通訊系統 (II)

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Chapter 8
Multichannel Modulation

Capacity of AWGN Channel

Capacity of AWGN Channel

- According to **Shannon's information capacity law**, the capacity of an AWGN channel is defined by

$$C = B\log_2\left[1 + P/\left(N_0 B\right)\right] = B\log_2\left[1 + \mathrm{SNR}\right] \quad \text{bits/sec}$$

  - where B is the channel bandwidth in hertz and SNR is measured at the **channel output**

Ch. 7

- Equivalently, the capacity C in bits per channel use is

Receive power **Receive SNR**

$$C' = \frac{1}{2}\log_2\left(1 + P/\sigma^2\right) = \frac{1}{2}\log_2\left(1 + \mathrm{SNR}\right) \quad \text{bits/transmission}$$

- In practice, we usually find that a **physically realizable encoding system** must transmit data at a rate R **less than** the maximum possible rate C for **reliable** reception.

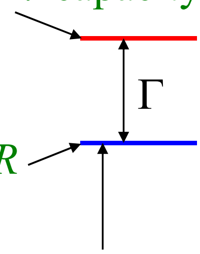
Signal-to-Noise Ratio Gap

- For an **implementable** system operating at a certain low enough probability of symbol error
  - We introduce a **signal-to-noise ratio gap** (or just **gap**), denoted by Γ, which is defined by

$$\Gamma = \frac{2^{2C} - 1}{2^{2R} - 1} = \frac{\mathrm{SNR}}{2^{2R} - 1}$$

  - C: the capacity of the **ideal encoding system**
  - R: the capacity of the corresponding **implementable encoding system**

Actual SNR ⇒ capacity C



Attainable capacity R ⇒ equivalent SNR

Depends on the encoding system

- It is a function of the **permissible probability of symbol error** P_e and the **encoding system** of interest
- It provides a measure of the "**efficiency**" of an encoding system
  - with respect to the **ideal transmission system**

Signal-to-Noise Ratio Gap (Cont.)

- A **small** (**large**) gap corresponds to an **efficient** (**inefficient**) encoding system
- Then, we have the attainable transmit data rate

$$R = \frac{1}{2}\log_2\left(1 + \mathrm{SNR}/\Gamma\right) \quad \text{bits/transmission}$$

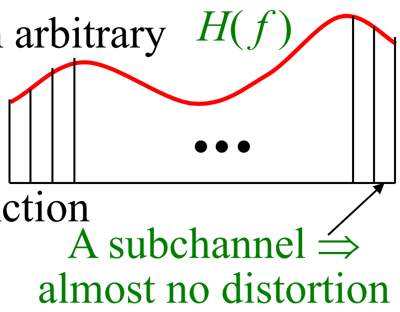
- For example: the desired **probability of symbol error** $P_e = 10^{-6}$
  - For an uncoded PAM or QAM system, the gap is **8.8 dB**
  - Through the use of channel coding (e.g., trellis codes), the gap may be reduced to as low as **1 dB**
- Because SNR = P/N_0B, the attainable **data rate** is defined as

$$R = \frac{1}{2}\log_2\left(1 + \frac{P}{\Gamma N_0 B}\right) \quad \text{bits/transmission}$$

More power is required

Continuous-Time Channel Partitioning

Continuous-Time Channel Partitioning



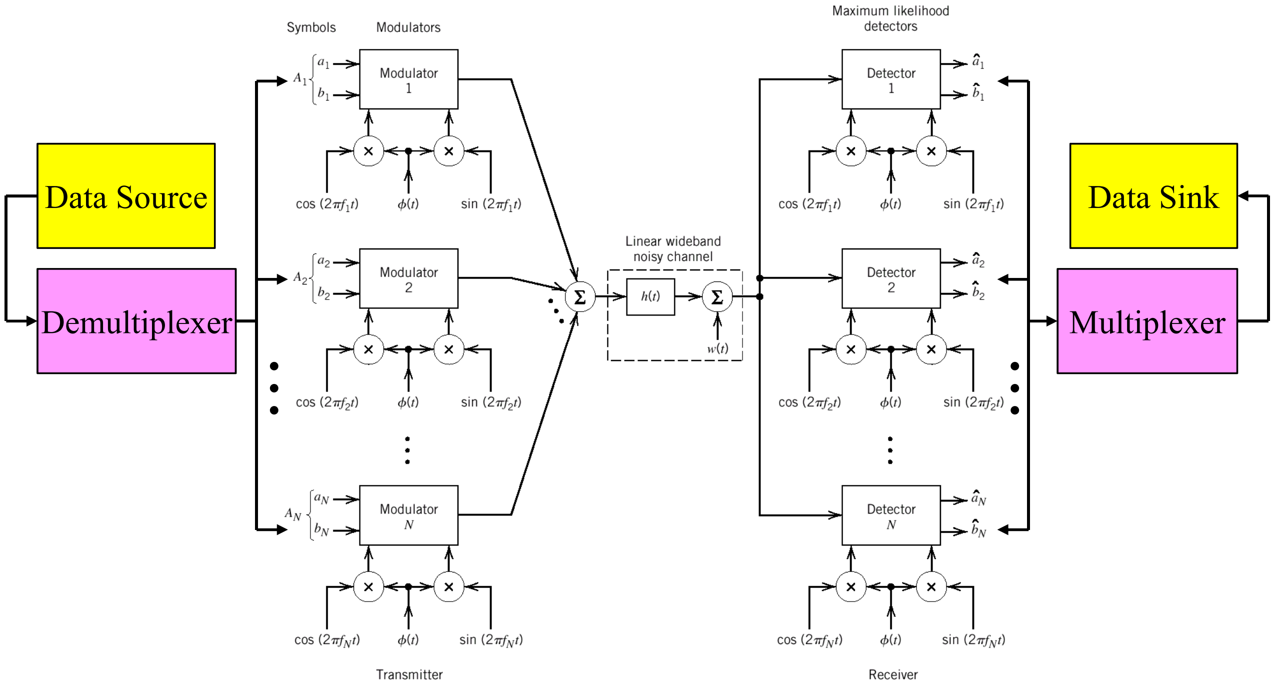
- Consider a linear **wideband channel** with an arbitrary frequency response $H(f)$.
  - Used as a single channel → distortion
  - $|H(f)|$ is approximated by a **staircase** function
  - Δf: the width of each **subchannel**
- In each step, the channel may be assumed to operate as an AWGN channel **free from inter-symbol interference**.
  - Transmitting a **wideband signal** is transformed into the transmission of a set of **narrowband orthogonal signals**
  - Each orthogonal **narrowband signal**, with **its own carrier**, is generated using a modulation technique, e.g., M-ary QAM
    - AWGN is the only transmission impairment (with a **constant response** for each subchannel) → no distortion

Continuous-Time Channel Partitioning (Cont.)

- Data transmission over each subchannel can be **optimized** by invoking **Shannon's information capacity law**
  - The optimization of each subchannel is performed **independently** of all the others
- The need for **complicated equalization** of a **wideband channel** (because of the **non-constant** response) is replaced by
  - The need of **demultiplexing** and **multiplexing**
    - **Demultiplexing**: Demultiplex the incoming data stream into multiple subchannels
    - **Multiplexing**: Multiplex the demodulated data from multiple subchannels to a single data stream

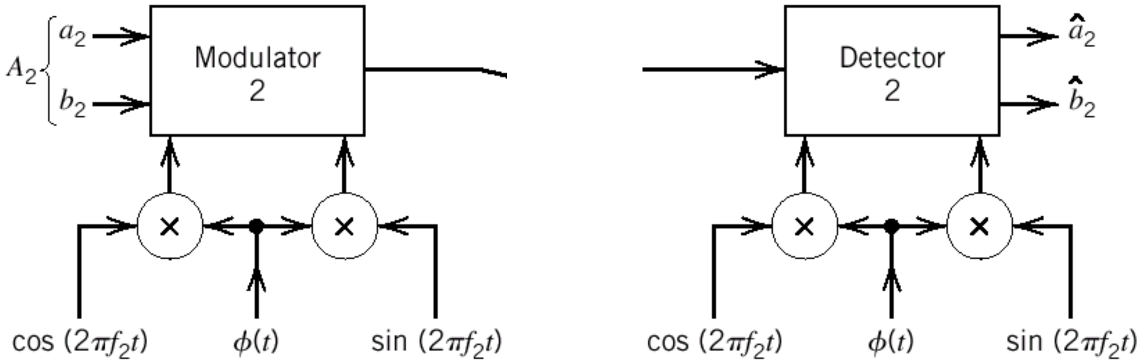
Continuous-Time Channel Partitioning (Cont.)

- A block diagram of the **multichannel** data transmission system



Continuous-Time Channel Partitioning (Cont.)

- The incoming data stream is first applied to a **demultiplexer**
  - Produce a set of N **substreams**
  - Each substream represents a sequence of **two-element** subsymbols, (a_n, b_n), $n = 1, 2, \cdots, N$, for **QAM modulation**
- The detected data of the N substreams are finally applied to a **multiplexer** to restore an output data stream



Geometric Signal-to-Noise Ratio

- In the **multichannel** transmission system, **each subchannel** is characterized by an SNR of its own.
  - However, it is highly desirable to derive a **single performance measure** of the **entire system**
- We assume that all of the subchannels are represented by one-dimensional constellations
  - The average channel capacity is

$$\boxed{C' = \frac{1}{2}\log_2\left(1 + P/\sigma^2\right)}$$

$$R = \frac{1}{N}\sum_{n=1}^{N} R_n = \frac{1}{2N}\sum_{n=1}^{N}\log_2\left(1 + \frac{P_n}{\Gamma\sigma_n^2}\right) = \frac{1}{\underline{2N}}\log_2\left[\prod_{n=1}^{N}\left(1 + \frac{P_n}{\Gamma\sigma_n^2}\right)\right]$$

Receive power

$$= \frac{1}{2}\log_2\left[\prod_{n=1}^{N}\left(1 + \frac{P_n}{\Gamma\sigma_n^2}\right)\right]^{\underline{1/N}}$$

bits/transmission

Geometric Signal-to-Noise Ratio (Cont.)

- Let $(\mathrm{SNR})_{\mathrm{overall}}$ denote the **overall SNR** of the entire system.
  - Then, we may express the rate R as

$$R = \frac{1}{2}\log_2\left[1+\frac{(\mathrm{SNR})_{\mathrm{overall}}}{\Gamma}\right] \quad \text{bits/transmission}$$

- Accordingly, the overall SNR is

$$(\mathrm{SNR})_{\mathrm{overall}} = \Gamma\left[\prod_{n=1}^{N}\left(1+\frac{P_n}{\Gamma\sigma_n^2}\right)^{1/N} - 1\right]$$

- If the SNR is large enough, we have the approximation

$$(\mathrm{SNR})_{\mathrm{overall}} \approx \prod_{n=1}^{N}\left(\frac{P_n}{\sigma_n^2}\right)^{1/N}$$

  - It is the **geometric mean** of the SNRs of the individual subchannels and is **independent** of the gap Γ.

Loading of the Multichannel Transmission System

Power Loading

- Define the magnitude response $g_n = |H(f_n)|$, $n = 1, 2, \cdots, N$
- Assuming that the number of subchannels N is large enough
  - g_n is a **constant** over the entire bandwidth Δf
  - The average channel capacity is

Transmit power

Receive power: $g_n^2 P_n$

$$R = \frac{1}{2N} \sum_{n=1}^{N} \log_2 \left(1 + \frac{g_n^2 P_n}{\Gamma \sigma_n^2} \right) \quad \text{bits/transmission}$$

  - where g_n and Γ are usually fixed; noise variance is $\Delta f N_0$, $\forall n$
- Goal: Optimize the overall bit rate R through a proper allocation of the **total transmit power** among the various subchannels
  - Subject to the total transmit power constraint

$$P = \sum_{n=1}^{N} P_n$$

Power Loading (Cont.)

- **Maximize** the bit rate R through an **optimal sharing** of the total transmit power P between the N subchannels
  - Subject to the total transmit power constraint P
- Through the **method of Lagrange multipliers**, the solution to the **constrained optimization problem** is

$$P_n + \frac{\Gamma \sigma_n^2}{g_n^2} = K, \quad n = 1, 2, \cdots, N$$

gain and noise power

$$\boxed{S_X(f_k) = K - \frac{S_N(f_k)}{\left|H(f_k)\right|^2}}$$

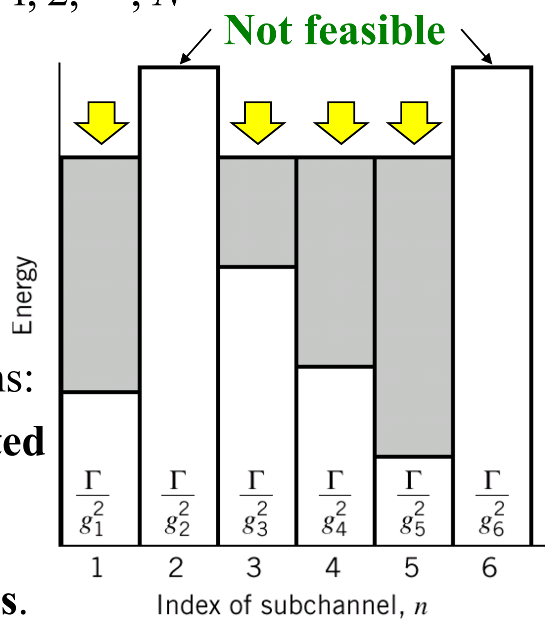
  - where K is a prescribed constant to meet the total transmit power constraint P
- The process of allocating the transmit power P to the individual subchannels is called **loading**.

Water-Filling Interpretation

- The **optimal** power allocation must satisfy the condition

$$P_n + \frac{\Gamma \sigma_n^2}{g_n^2} = K, \quad n = 1, 2, \cdots, N$$

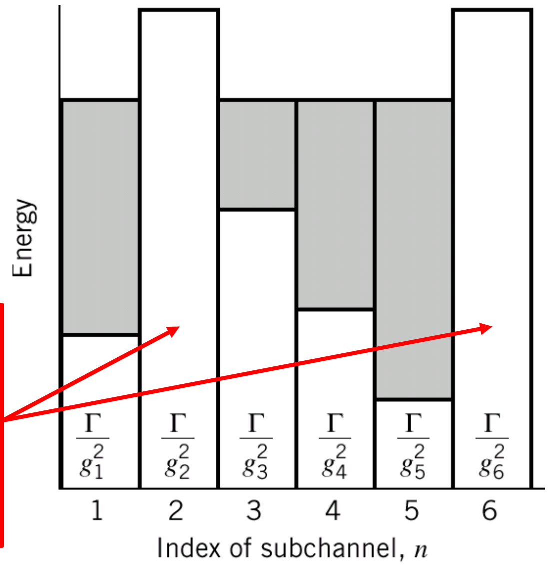
- Consider the case with $N = 6$
  - The gap Γ is assumed to be constant over all subchannels
  - The average noise power is set to $\sigma_n^2 = N_0 \Delta f = 1$
- We make the following observations:
  - With $\sigma_n^2 = 1$, the sum of **allocated power** P_n and the **scaled noise power** Γ/g_n^2 is equal to a constant K for **four subchannels**.



Water-Filling Interpretation (Cont.)

- The sum of power allocations to these four subchannels consumes **all the available transmit power** P.
- The remaining two subchannels have been eliminated from consideration
  - Because they would each require **negative power** to satisfy the condition (i.e., $P_n < 0$)

The channel response g_n is **too small**
$\Rightarrow$ The **scaled noise power** Γ/g_n^2 is **very large**
$\Rightarrow$ Allocating power to these two channels is **inefficient**

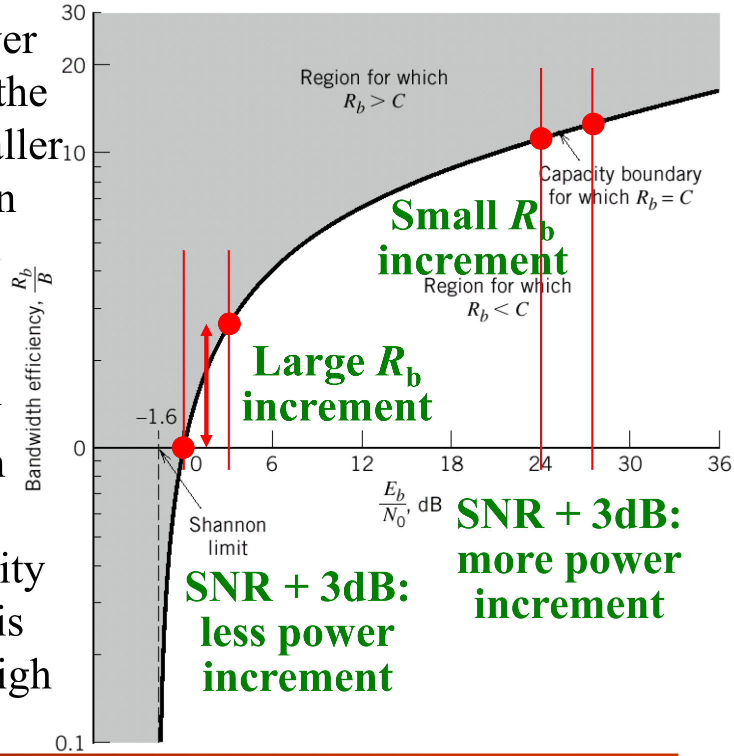


Water-Filling Interpretation (Cont.)

- The optimum solution for **loading** is referred to as **water-filling solution**
- This terminology follows from analogy of our optimization problem with
  - **A fixed amount of water**—standing for transmit power
  - Being **poured into a container** with a number of connected regions
  - **Each having a different depth**—standing for noise power
- In such a scenario, the water distributes itself in such a way that
  - A **constant water level** is attained across the whole container, hence the term “**water filling**”

Water-Filling Interpretation (Cont.)

- If the same amount of power is added, the increment in the capacity will be larger/smaller in the low/high SNR region
- For example, if the SNR is increased by 3 dB
  - The required additional power is less in the low SNR region than that in the high SNR region
  - The increment in capacity in the low SNR region is larger than that in the high SNR region



Process of Loading

- The allocation of the fixed transmit power P among the various subchannels can be formularized as follows:
  - There are a total of $(N+1)$ unknowns and $(N+1)$ equations

$$P_n + \frac{\Gamma \sigma_n^2}{g_n^2} = K, \quad n = 1, 2, \cdots, N$$

$$\sum_{i=1}^{N} P_n = P$$

Unknowns: P_n and K

$$\begin{bmatrix} 1 & 1 & \cdots & 1 & 0 \\ 1 & 0 & \cdots & 0 & -1 \\ 0 & 1 & \cdots & 0 & -1 \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & \cdots & 1 & -1 \end{bmatrix} \begin{bmatrix} P_1 \\ P_2 \\ P_3 \\ \vdots \\ K \end{bmatrix} = \begin{bmatrix} P \\ -\Gamma\sigma^2/g_1^2 \\ -\Gamma\sigma^2/g_2^2 \\ \vdots \\ -\Gamma\sigma^2/g_N^2 \end{bmatrix} \Rightarrow \mathbf{Mu} = \mathbf{c}$$

Process of Loading (Cont.)

- Multiplying the inverse of **M** on both sides of the equation
  - The unknowns $P_1, P_2, \cdots, P_N$, and K can be obtained

$$\begin{bmatrix} P_1 \\ P_2 \\ P_3 \\ \vdots \\ K \end{bmatrix} = \mathbf{u} = \mathbf{M}^{-1}\mathbf{c}$$

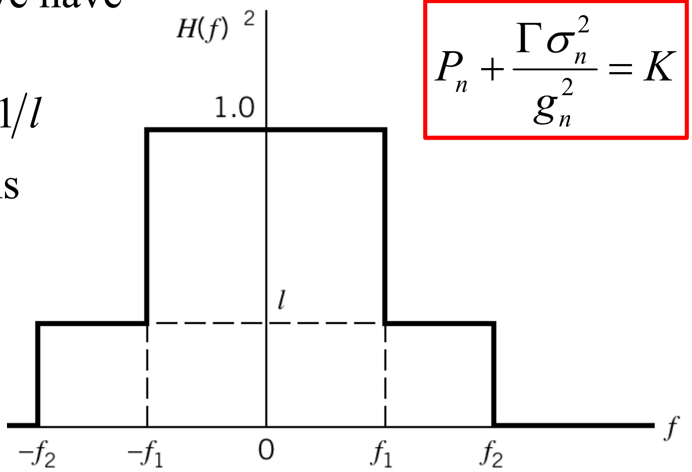
  - K is always positive
  - It is possible for some of the P_n values to be **negative**
    - In such a situation, the solution is **incorrect**
    - **Discard** the subchannels with negative P_n values, and **resolve** the problem with reduced number of subchannels
  - If all the P_n values are **positive**, the solution is **correct**

Example

- Consider a linear channel whose squared magnitude response $|H(f)|^2$ has the **piecewise linear form**
- To simplify the example, we have set the gap $\Gamma = 1$ and the noise variance $\sigma_n^2 = 1$
- Under this set of values, we have

$$P_1 + P_2 = P$$
$$P_1 - K = -1; \quad P_2 - K = -1/l$$

$$\boxed{P_n + \frac{\Gamma\sigma_n^2}{g_n^2} = K}$$



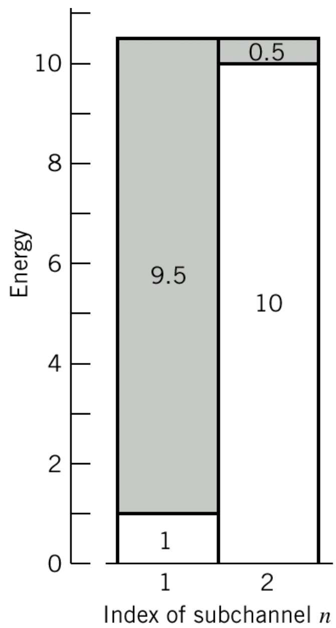
- Solving the three equations for P_1, P_2, and K

$$P_1 = (P - 1 + 1/l)/2$$
$$P_2 = (P + 1 - 1/l)/2$$
$$K = (P + 1 + 1/l)/2$$

Example (Cont.)

- Since $0 < l < 1$, it follows that $P_1 > 0$
- But it is possible for P_2 to be **negative**
  - It happens if $l < 1/(P+1)$
  - Correspondingly, P_1 exceeds the transmit power P ($P_1 > P$)
- Therefore, it follows that, in this example, the only acceptable solution is to have $1/(P+1) < l < 1$.
- Let $P = 10$ and $l = 0.1$
  - The desired solution is

$$P_1 = 9.5$$
$$P_2 = 0.5$$
$$K = 10.5$$



Orthogonal Frequency Division Multiplexing (OFDM)

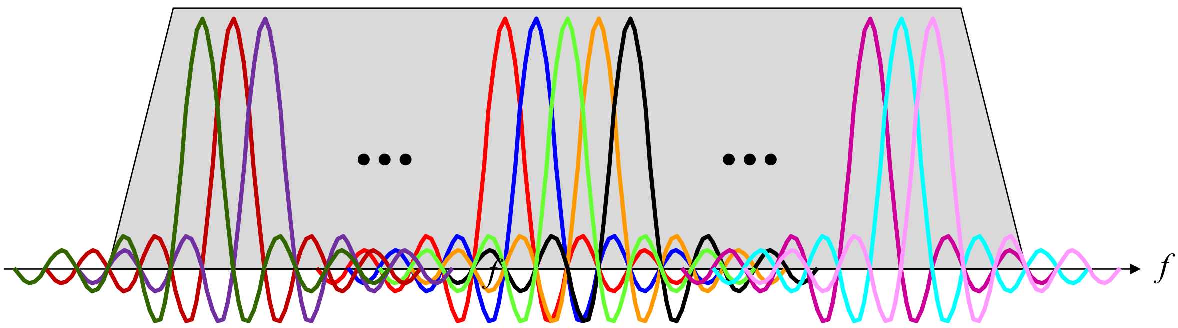
OFDM Concept

- **Orthogonal frequency division multiplexing** (**OFDM**) is a form of **multi-carrier modulation**.
  - OFDM is particularly well suited for **high data-rate transmission** over **delay-dispersive channels**.
- Specifically, a large number of closely spaced **orthogonal subcarriers (tones)** is used to support the transmission.
  - The incoming data stream is divided into a number of **low data-rate sub-streams**, one for each subcarrier
- In addition, two other changes have to be made for OFDM:
  - In the transmitter, an **upconverter** is included after the DAC to **translate** the signal to the **transmission band**
  - In the receiver, a **downconverter** is included before the ADC to **translate** the signal to the **baseband**

OFDM Concept (Cont.)

- **Orthogonal frequency division multiplexing (OFDM)** is a promising technique because of its
  - High bandwidth efficiency and
  - Resistance to multipath fading
- **Orthogonality** is maintained among the subcarriers
- **Narrowband** transmission for each digitally modulated signal

Transmission Band

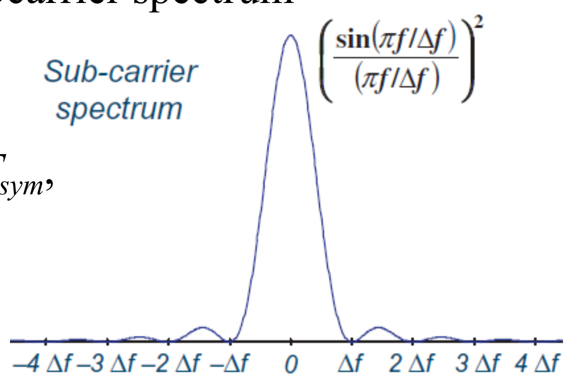


OFDM Concept (Cont.)

- However, the following characteristics distinguish OFDM from a straightforward multi-carrier extension:
  - The use of a typically **very large number** of relatively narrowband subcarriers (e.g., several hundred subcarriers)
  - Simple **rectangular pulse shaping** (time-domain) is used

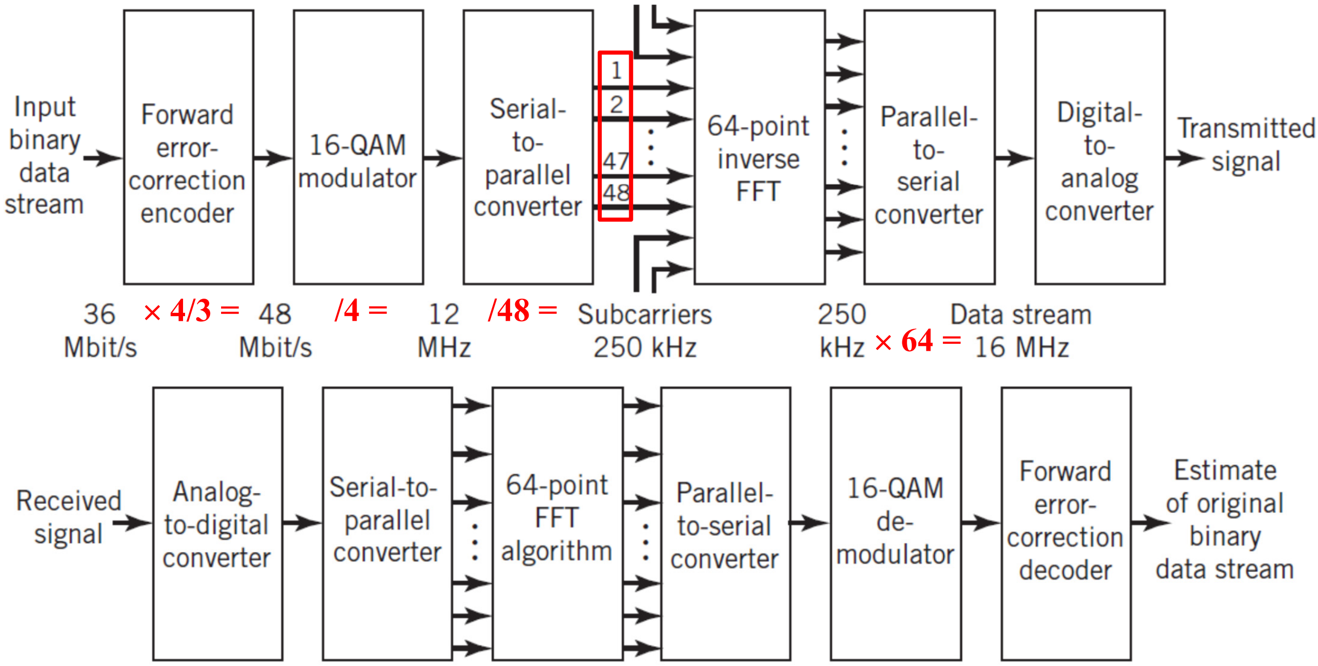
  $\Rightarrow$ A sinc-square-shaped per-subcarrier spectrum
  - **Tight frequency-domain packing** of the subcarriers

  $\Rightarrow$ A subcarrier spacing $\Delta f = 1/T_{sym}$, T_{sym} is the per-subcarrier modulation-symbol duration



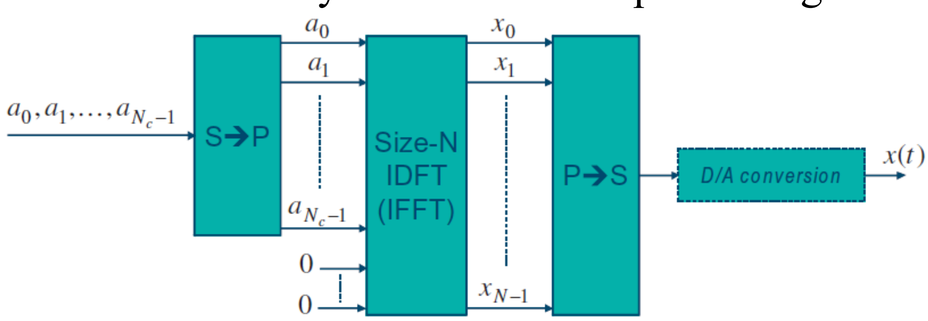
OFDM Transmitter/Receiver

- Block diagrams of transmitter/receiver for a 36 Mbits/s system

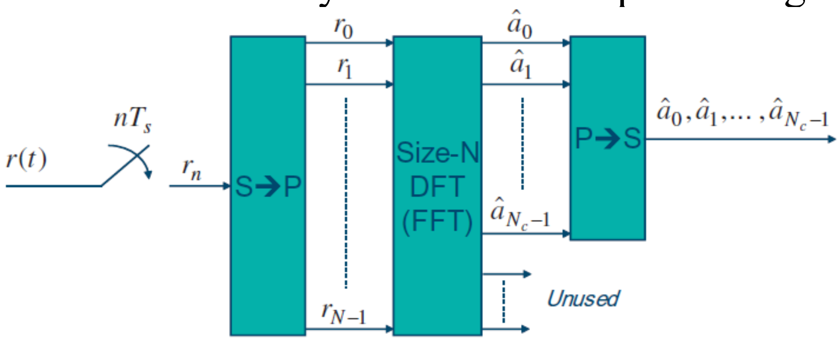


OFDM Implementation

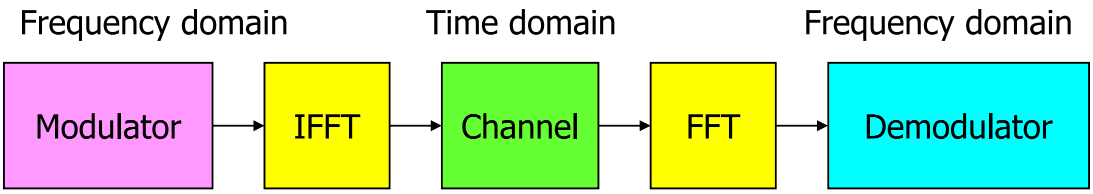
- OFDM **modulation** by means of **IFFT** processing

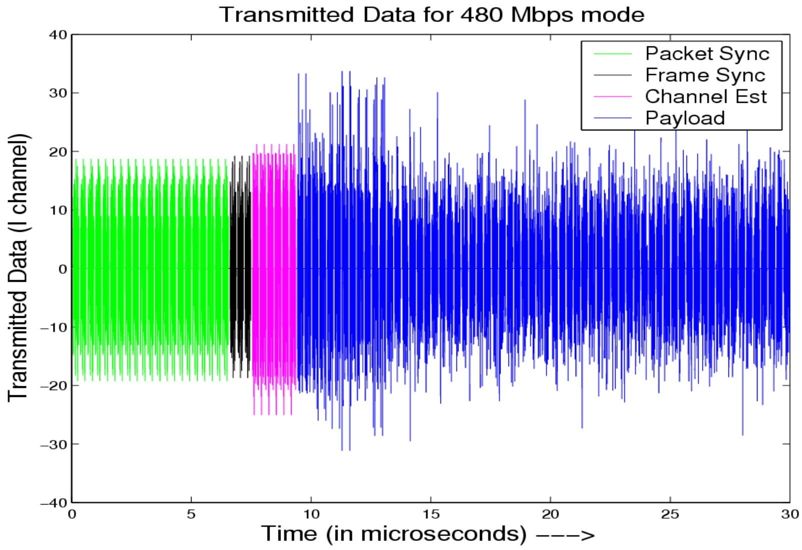


- OFDM **demodulation** by means of **FFT** processing



OFDM Transmission





Homework

- **You must give detailed derivations or explanations, otherwise you get no points.**
- Communication Systems, Simon Haykin **(4th Ed.)**
- 6.43;