通訊系統 (II)

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Chapter 3
Hybrid Amplitude/Phase Modulation

Introduction

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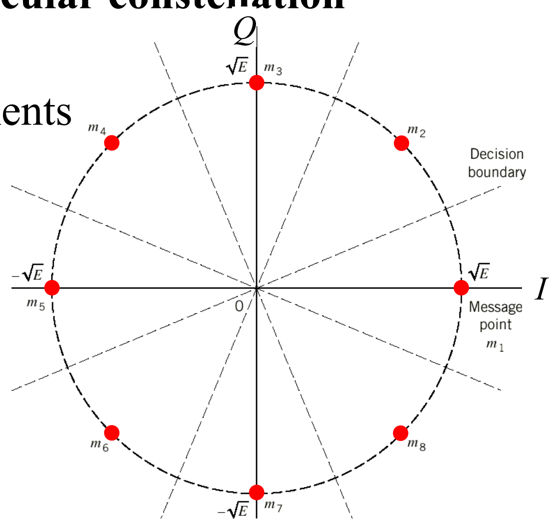
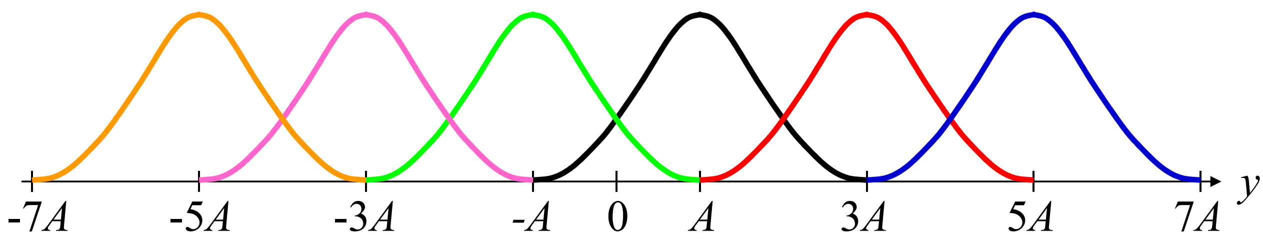
- In an *M*-ary PSK system, the **in-phase** and **quadrature** components are **inter-related**
  - The envelope is constrained to remain **constant**
  - The message points forms a **circular constellation**
- If the constraint is removed: the **in-phase** and **quadrature** components are permitted to be **independent**
  - A new modulation: *M*-ary **quadrature amplitude modulation (QAM)**
  - The carrier experiences **amplitude** as well as **phase** modulation



M-ary Amplitude-Shift Keying (ASK)

Baseband *M*-ary Pulse-Amplitude Modulation

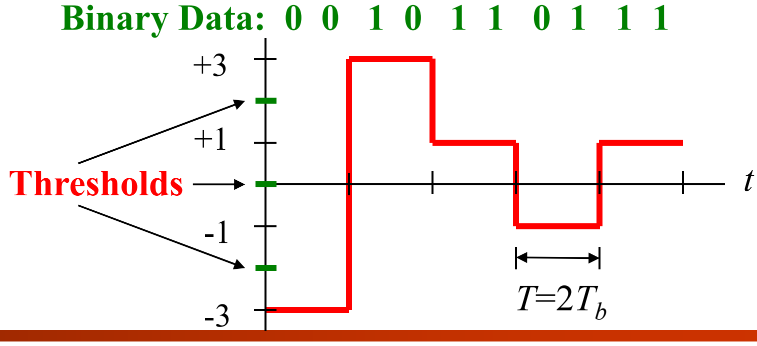
- In a **baseband** *M*-ary **Pulse-Amplitude Modulation** (**PAM**) system, the modulator produces one of *M* possible **amplitude levels** with $M > 2$
  - Each symbol contains **$\log_2 M$ bits** of data
  - A symbol is represented by a specified **amplitude level**
- To realize **the same average probability of symbol error**
  - The amplitude levels are $\pm A, \pm 3A, \pm 5A, \ldots, \pm(M-1)A$
  - The maximum amplitude level is **about MA** for $M >> 2$



Bandpass Amplitude-Shift Keying (ASK)

- The **bandpass** digital *M*-ary PAM is also called *M*-ary **Amplitude-Shift Keying** (**ASK**).
- The mapping or assignment of $k = \log_2 M$ information bits to the $M = 2^k$ possible signal amplitudes can be done in many ways
  - In demodulation, the most likely errors caused by noise involve the erroneous selection of an **adjacent** amplitude
  - **Gray encode**: the adjacent signal amplitudes differ by one bit

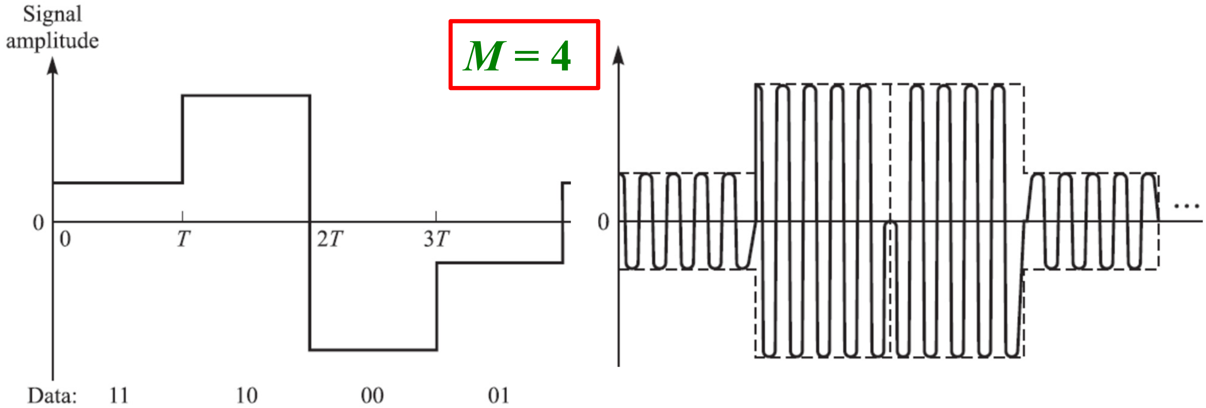
Symbol	Amplitude
10	+3
11	+1
01	–1
00	–3



Signal Space of *M*-ary ASK

- In the **on–off keying** (OOK) version of an ASK system
  - Symbol 1 is represented by transmitting a sinusoidal carrier
  - Symbol 0 is represented by switching off the carrier
- There is only **one basis function** of unit energy

$$\phi_1(t) = \sqrt{2/T}\cos(2\pi f_c t),\ 0 \le t < T$$



M-ary Quadrature Amplitude Modulation (QAM)

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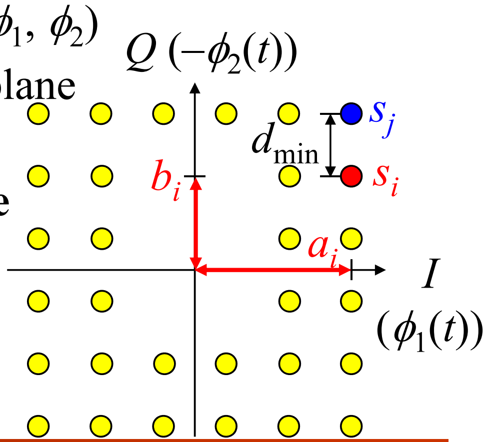
- *M*-ary **Quadrature Amplitude Modulation** (**QAM**) is a **two-dimensional** generalization of *M*-ary PAM
- The two **orthonormal basis functions** are

$$\phi_1(t) = \sqrt{2/T}\cos(2\pi f_c t),\ 0 \le t < T; \quad \phi_2(t) = \sqrt{2/T}\sin(2\pi f_c t),\ 0 \le t < T$$

  - A symbol is mapped to a signal point on the **two-dimensional** plane constructed by (ϕ_1, ϕ_2)
- Let the message point $\mathbf{s}_i$ in the (ϕ_1, ϕ_2) plane be denoted by ($a_i d_{\min}/2$, $b_i d_{\min}/2$)
  - where $d_{\min}$ is the **minimum distance** between any two message points
  - a_i and b_i are **integers**, for i = 1, 2, …, M



Signal Space of M-ary QAM

- Let $d_{min} = 2\sqrt{E_0}$, where E_0 is the **one-dimension energy** of the signals with the **lowest amplitude** ($a_i = \pm 1$ or $b_i = \pm 1$)
- The transmitted M-ary QAM signals are defined by

$$s_k(t) = \sqrt{\frac{2E_0}{T}} a_k \cos(2\pi f_c t) - \sqrt{\frac{2E_0}{T}} b_k \sin(2\pi f_c t), \quad 0 \le t < T$$

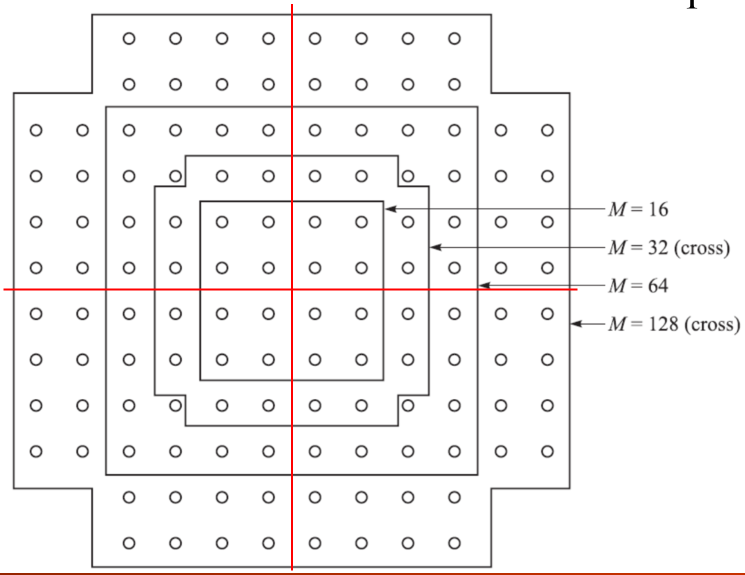
  – **Two phase-quadrature carriers:** $\cos(2\pi f_c t)$ and $\sin(2\pi f_c t)$

  – **Amplitude modulation:** a_k and b_k

  ⇒ **Quadrature Amplitude Modulation**

M-ary QAM Constellations

- There are two distinct QAM constellations:
  – **Square constellations:** an **even** number of bits per symbol
  – **Cross constellations:** an **odd** number of bits per symbol



M-ary QAM Signals: Square Constellations

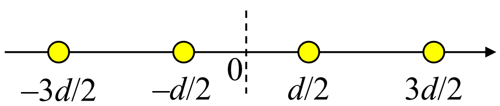
- With an even number of bits per symbol, we have $L = \sqrt{M}$
- An M-ary QAM square constellation can always be viewed as the **Cartesian product** of a **one-dimensional *L*-ary PAM constellation** with itself
  - The set of **all possible ordered pairs of coordinates**
  - The **first** (**second**) coordinate is taken from the **first** (**second**) set

First set **Second set**

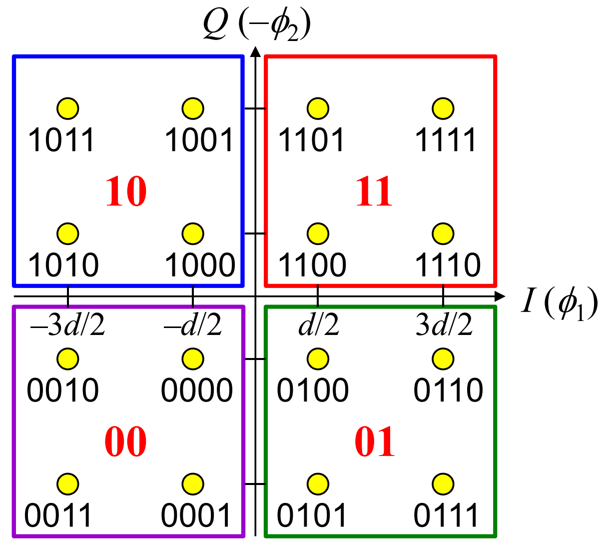
$$\{a_i, b_i\} = \begin{bmatrix} (-L+1, L-1) & (-L+3, L-1) & \cdots & (L-1, L-1) \\ (-L+1, L-3) & (-L+3, L-3) & \cdots & (L-1, L-3) \\ \vdots & \vdots & & \vdots \\ (-L+1, -L+1) & (-L+3, -L+1) & \cdots & (L-1, -L+1) \end{bmatrix}$$

Example 3

- Consider a 16-QAM constellation: 4 bits per symbol
  - The **left-most two** bits: a **quadrant** in the $\{\phi_1, \phi_2\}$-plane
  - The **right-most two** bits: one **signal point** in a quadrant
  - The data mapping of signal points still follows the **Gray coding rule**
  - Any two nearest neighbors differ in only **one bit**



One-dimensional 4-ary PAM constellation



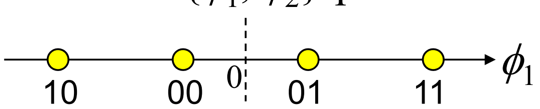
Example 3 (Cont.)

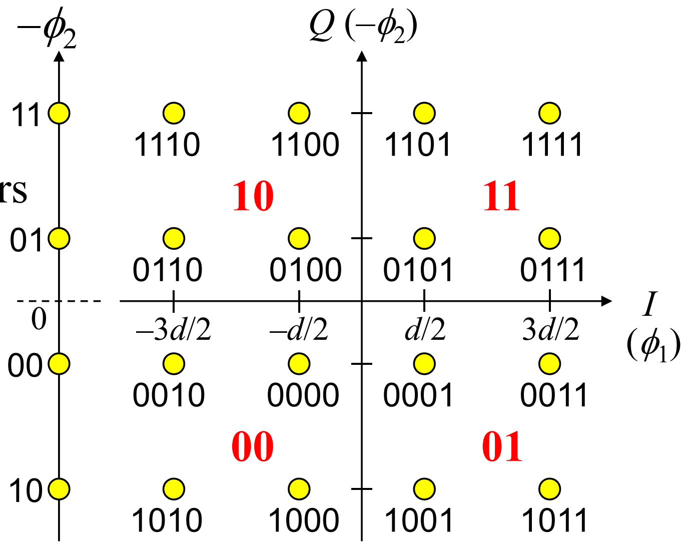
- The matrix of the two-dimensional coordinate is

$$\{a_i, b_i\} = \begin{bmatrix} (-3,+3) & (-1,+3) & (+1,+3) & (+3,+3) \\ (-3,+1) & (-1,+1) & (+1,+1) & (+3,+1) \\ (-3,-1) & (-1,-1) & (+1,-1) & (+3,-1) \\ (-3,-3) & (-1,-3) & (+1,-3) & (+3,-3) \end{bmatrix}$$

Another Square M-ary QAM Data Mapping

- Consider a 16-QAM constellation: 4 bits per symbol
- The data mapping in each of the **one-dimensional** 4-ary PAM constellation (ϕ_1 or ϕ_2) follows the **Gray coding rule**
  - In the 2D constellation, the data mapping follows the **Gray coding rule**
  - Any two nearest neighbors differ in only **one bit**
  - The **2nd and 4th bits** shows a **quadrant** in the $\{\phi_1, \phi_2\}$-plane





Error Probability of M-ary QAM: Square

- A QAM square constellation can be factored into the product of two PAM constellations
  - The **probability of correct detection** for M-ary QAM is

$$P_c = \left(1 - P_e'\right)^2$$

    - where P_e' is the **probability of symbol error** for the corresponding L-ary PAM constellation with $L = \sqrt{M}$

$$P_e' = \left(1 - 1/\sqrt{M}\right)\mathrm{erfc}\left(\sqrt{E_0/N_0}\right), \quad E_0 = \left(d_{\min}/2\right)^2$$

- The **probability of symbol error** for M-ary QAM is

$$P_e = 1 - P_c = 1 - \left(1 - P_e'\right)^2 \simeq 2P_e'$$

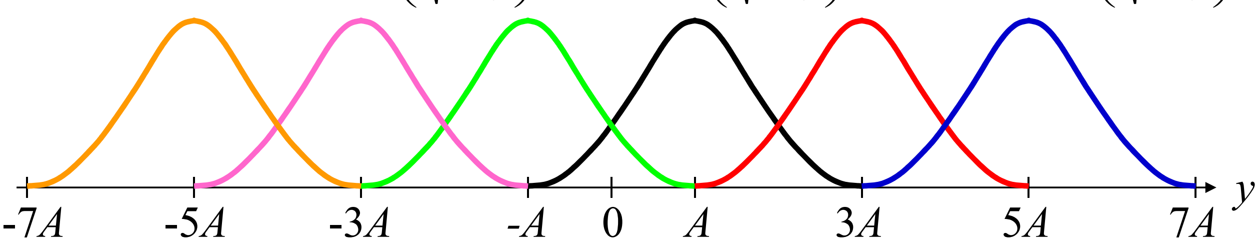
  - where it is assumed that P_e' is small enough
- Hence,

$$P_e \simeq 2\left(1 - \frac{1}{\sqrt{M}}\right)\mathrm{erfc}\left(\sqrt{\frac{E_0}{N_0}}\right)$$

Error Probability of M-ary QAM: Square (Cont.)

- The **probability of symbol error** for the corresponding L-ary PAM constellation with $L = \sqrt{M}$:
  - For binary PAM: One-sided error $P_e' = \frac{1}{2}\mathrm{erfc}\left(\sqrt{\frac{E_0}{N_0}}\right)$
  - For L-ary PAM:
    - Two-sided error: (L – 2) points
    - One-sided error: 2 points

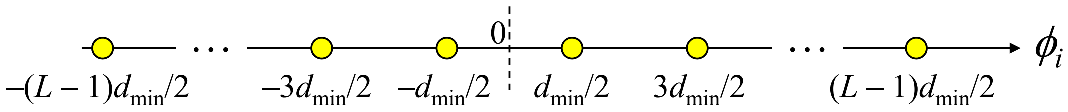
$$P_e' = \frac{2 \times (L-2)}{2L}\mathrm{erfc}\left(\sqrt{\frac{E_0}{N_0}}\right) + \frac{2}{2L}\mathrm{erfc}\left(\sqrt{\frac{E_0}{N_0}}\right) = \left(1 - \frac{1}{L}\right)\mathrm{erfc}\left(\sqrt{\frac{E_0}{N_0}}\right)$$



Symbol Energy of *M*-ary QAM: Square

- The transmitted symbol energy in *M*-ary QAM is variable
  - Depending on the transmitted **data** (the **signal point**)
  - For example, in the 16-QAM constellation
    - $2E_0$ (E_0+E_0): 4 signal points
    - $10E_0$ ($9E_0+E_0$): 8 signal points
    - $18E_0$ ($9E_0+9E_0$): 4 signal points
- The average symbol energy in 1D *L*-ary PAM constellation is
  - Assuming that the *L* levels are equiprobable

$$E'_{av} = \frac{2}{L} \times \left[E_0 + 9E_0 + 25E_0 + \cdots + (L-1)^2 E_0 \right] = \frac{2E_0}{L} \sum_{i=1}^{L/2} (2i-1)^2$$



Symbol Energy of *M*-ary QAM: Square (Cont.)

- The average symbol energy in the *M*-ary QAM constellation is
  - The sum of the energy of the two PAM constellations

$$E_{av} = 2 \times E'_{av} = \frac{4E_0}{L} \sum_{i=1}^{L/2} (2i-1)^2$$

$$= \frac{2(L^2-1)E_0}{3} = \frac{2(M-1)E_0}{3}$$

- The **probability of symbol error** for *M*-ary QAM can be expressed as a function of E_{av}

$$P_e \simeq 2\left(1 - \frac{1}{\sqrt{M}}\right) \mathrm{erfc}\left(\sqrt{\frac{3E_{av}}{2(M-1)N_0}}\right)$$

- For $M = 4$ (i.e., QPSK)

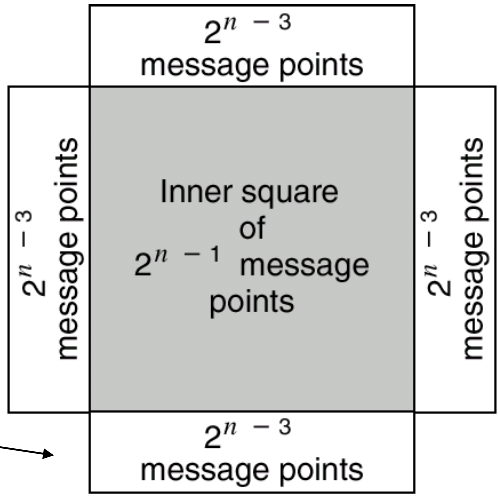
$$P_e \simeq \mathrm{erfc}\left(\sqrt{E_{av}/2N_0}\right) = \mathrm{erfc}\left(\sqrt{E/2N_0}\right)$$

M-ary QAM Signals: Cross Constellations

- When there is an odd number of bits per symbol (n) in M-ary QAM, a **cross constellation** is used
  - Start with a **square** constellation with ***n* – 1** bits per symbol
  - Extend **each side** of the square constellation by **adding** 2^{n-3} signal points
  - Ignore the **corners** in the extension

    $2^n = 2^{n-1} + 4 \times 2^{n-3} = 2^{n-1} + 2^{n-1}$

    $2^{n-3} = 2^{(n-1)/2} \times 2^{(n-5)/2}$
- It is **not possible** to perfectly Gray code a cross constellation



Ignore to reduce average symbol energy

Error Probability of M-ary QAM: Cross

- Unlike the square constellations, a **cross constellation cannot** be expressed as the product of two 1D PAM constellations
- The determination of the symbol error probability for M-ary QAM with a cross constellation is complicated
- The **symbol error probability** for a cross constellation can be approximated as the result for a square constellation
  - Approximated as a square constellation with $M' = 2M$

$$P_e \simeq 2\left(1 - \frac{1}{\sqrt{2M}}\right)\text{erfc}\left(\sqrt{\frac{E_0}{N_0}}\right), \quad \text{Cross Constellation}$$

$$P_e \simeq 2\left(1 - \frac{1}{\sqrt{M}}\right)\text{erfc}\left(\sqrt{\frac{E_0}{N_0}}\right), \quad \text{Square Constellation}$$

$M' = 32$
$M = 16$

Homework

- **You must give detailed derivations or explanations, otherwise you get no points.**
- Communication Systems, Simon Haykin **(4th Ed.)**
- 6.15;
- 6.16;