無線通訊系統
(Wireless Communications Systems)

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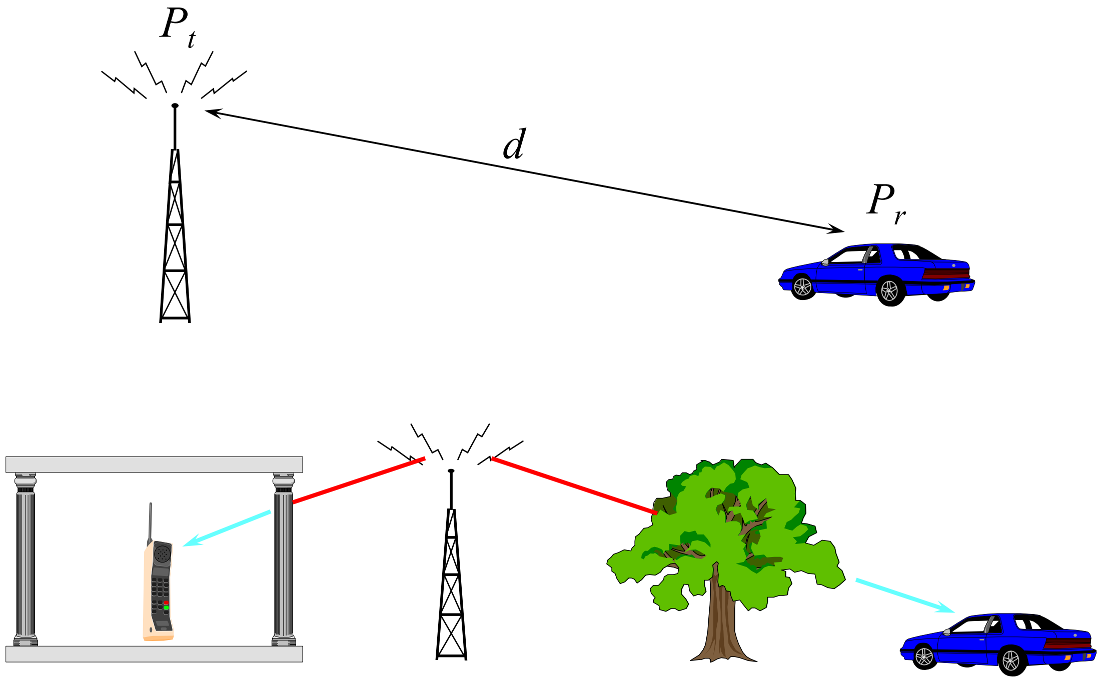
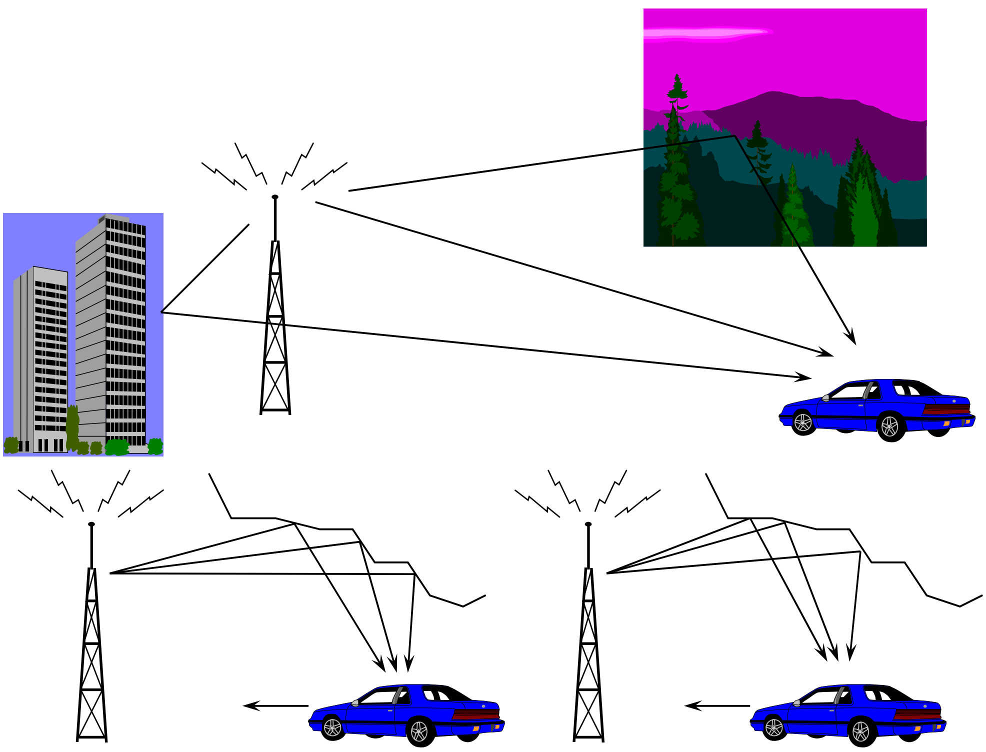
Chapter 2
Propagation Effects

Path Loss and Shadowing

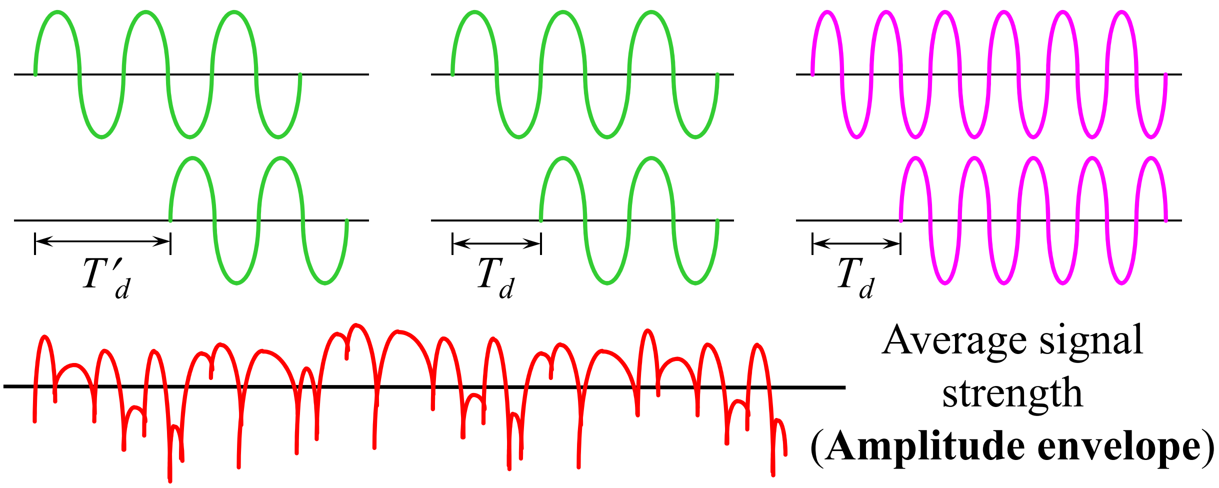


Multipath Propagation

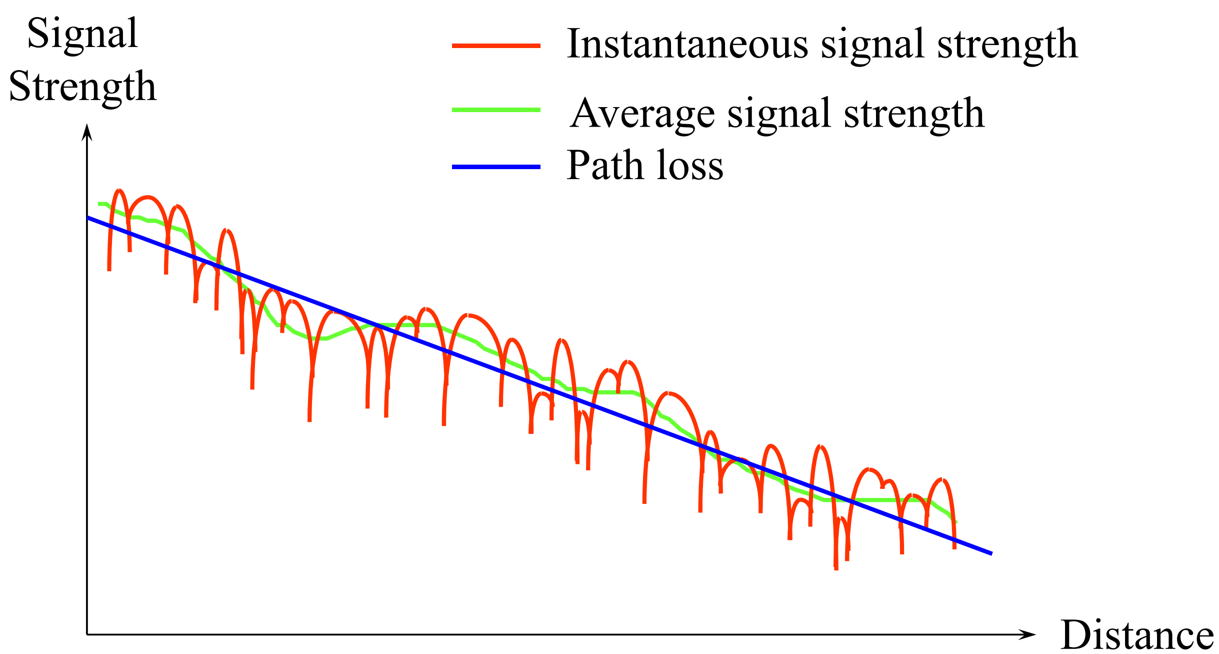


Fast Multipath Fading

- The variation of propagation channel results in the change of the received signal strength
- For the same propagation environment, **different frequency components** may experience different fading characteristics



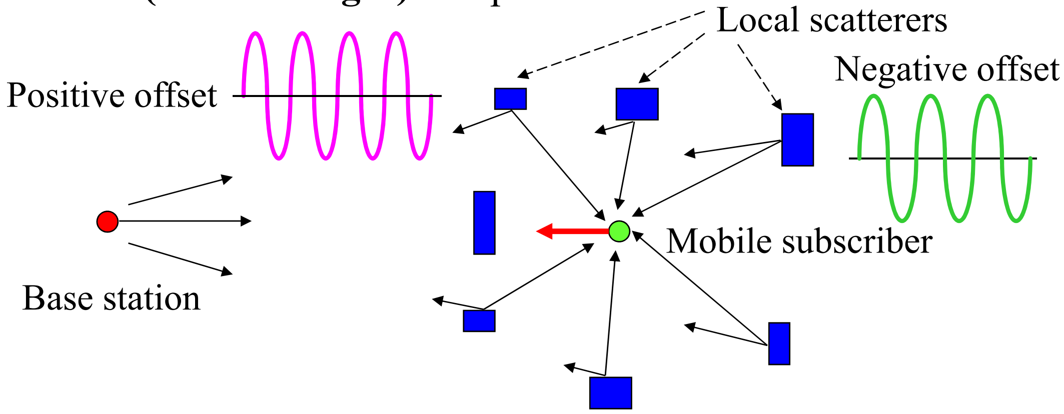
Radio Propagation



Basic Concepts of Propagation Modeling

Radio Propagations

- **Reciprocity Theorem :** If a propagation path exists, it carries energy equally well in **both directions**
- An MS in a typical macrocellular environment is usually surrounded by local scatterers
  - The plane waves arrive from many directions without a direct **LOS (Line-Of-Sight)** component



Radio Propagations (Cont.)

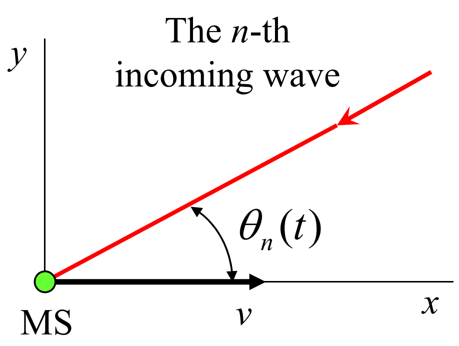
- **MS** in a **macrocellular** system: **isotropic scattering**
  - The arriving plane waves arrive from **all directions** with **equal probability**
  - In general, no direct LOS path exists between an MS and the BS
- **BS** in a **macrocellular** system: relatively free from local scatterers
  - The plane waves tend to arrive from **one general direction**
  - The cell radius is from 0.5km to several kilometers
- In a **microcellular** environment:
  - The BS antennas are only moderately elevated above the local scatterers
  - The cell radius is from 100m to several hundred meters
  - **A direct LOS path** may exist between an MS and the desired BS

Doppler (Frequency) Shift

- **Doppler (frequency) shift** is introduced for a mobile user
  - MS velocity: v
  - The **incidence angle** of the incoming wave: $\theta_n(t)$

$$f_{D,n}(t) = f_m \cos\theta_n(t) \quad \text{Hz}$$

  - where $f_m = v/\lambda_c$ and λ_c is the wavelength



Multipath Fading Channel

- Consider the transmission of the band-pass signal $s(t)$:

$$s(t) = \Re\left\{ \tilde{s}(t) e^{j2\pi f_c t} \right\}$$

  – $\tilde{s}(t)$ is the complex envelope and f_c is the carrier frequency

- The received band-pass signal is:

$$r(t) = \Re\left\{ \tilde{r}(t) e^{j2\pi f_c t} \right\}$$

  – $\tilde{r}(t)$ is the complex envelope

$$\boxed{\alpha_n(t);\ \tau_n(t);\ f_{D,n}(t)}$$

$$r(t) = \Re\left\{ \sum_{n=1}^{N} \alpha_n(t)\, e^{j2\pi\left(f_c + \underline{f_{D,n}(t)}\right)\left(t - \underline{\tau_n(t)}\right)}\ \tilde{s}(t-\tau_n(t)) \right\}$$

$$= \Re\left\{ \sum_{n=1}^{N} \alpha_n(t)\, e^{-j2\pi\left[\left(f_c + f_{D,n}(t)\right)\tau_n(t) - f_{D,n}(t)t\right]}\ \tilde{s}(t-\tau_n(t)) e^{j2\pi f_c t} \right\}$$

$$= \Re\left\{ \tilde{r}(t) e^{j2\pi f_c t} \right\}$$

Multipath Fading Channel (Cont.)

- The received complex low-pass signal (for an N-path channel):

$$\tilde{r}(t) = \sum_{n=1}^{N} \alpha_n(t)\, e^{-j2\pi\left[\left(f_c + f_{D,n}(t)\right)\tau_n(t) - f_{D,n}(t)t\right]}\ \tilde{s}(t-\tau_n(t))$$

  – $\alpha_n(t)$ is the amplitude gain and $\tau_n(t)$ is the time delay

$$\Rightarrow \tilde{r}(t) = \sum_{n=1}^{N} \alpha_n(t)\, e^{-j\phi_n(t)}\ \tilde{s}(t-\tau_n(t))$$

  – The phase associated with the n-th path is

$$\phi_n(t) = 2\pi\left[\left(f_c + f_{D,n}(t)\right)\tau_n(t) - f_{D,n}(t)t\right]$$

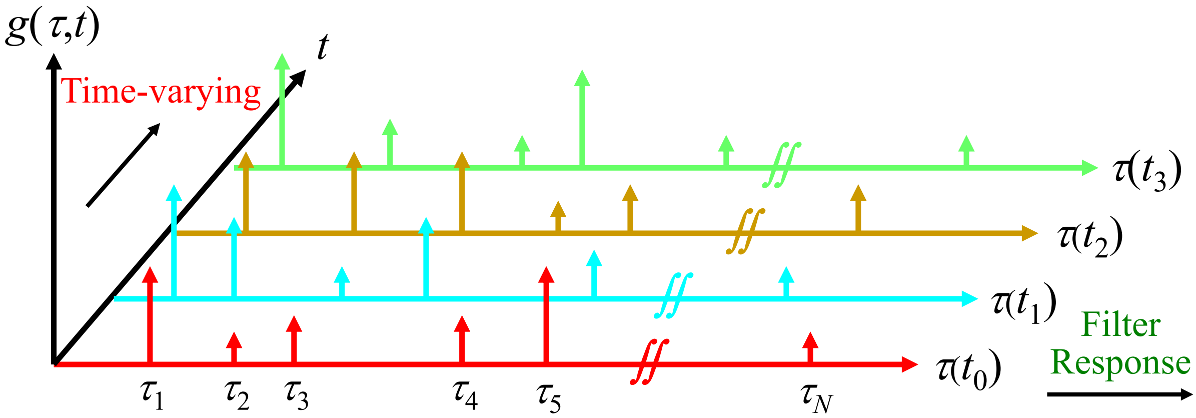
- The phase can be regarded as a **uniformly random** phase
  – Since $f_c \times \tau_n(t) >> 1$

Channel Modeling as a Filter

- The channel is modeled as a time-variant linear filter

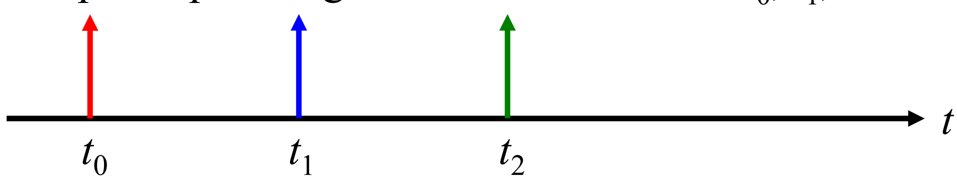
$$g(\tau,t)=\sum_{n=1}^{N}\alpha_n(t)e^{-j\phi_n(t)}\delta(\tau-\tau_n(t))$$

  - A small change in path delay $\tau_n(t)$ causes a large change in phase $\phi_n(t)$ (due to a very large $f_c+f_{D,n}(t)$)
  - Random **amplitude and phase** for each received path

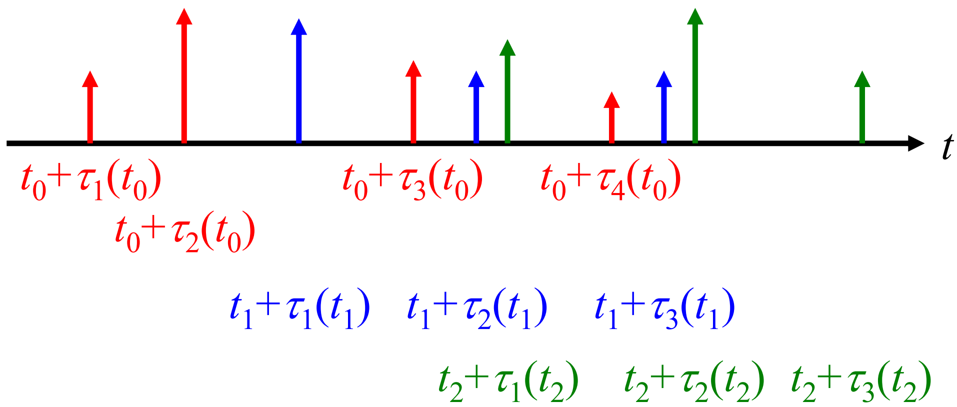


Channel Modeling as a Filter (Cont.)

- If multiple impulse signals are transmitted at t_0, t_1, …



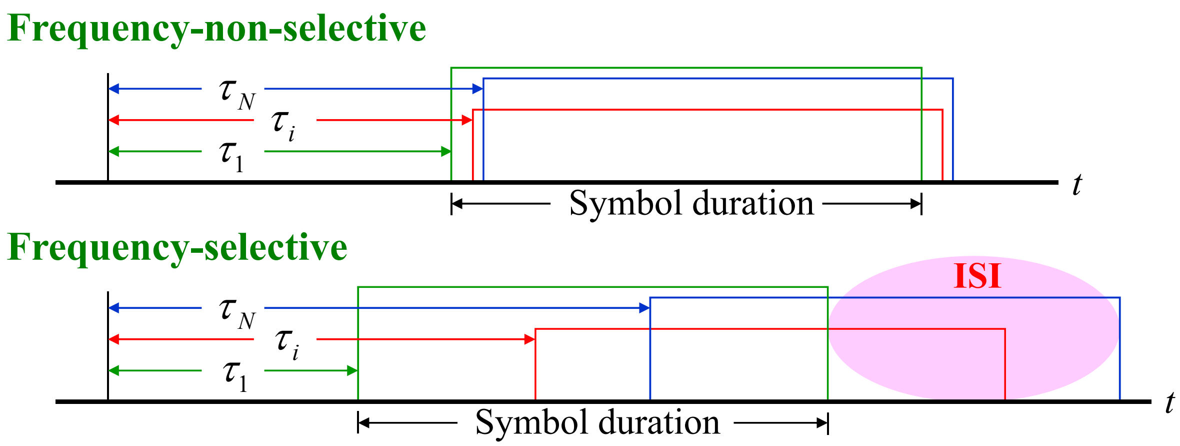
- The signal received at a receiver is



Freq.-Selective & -Non-Selective Fading

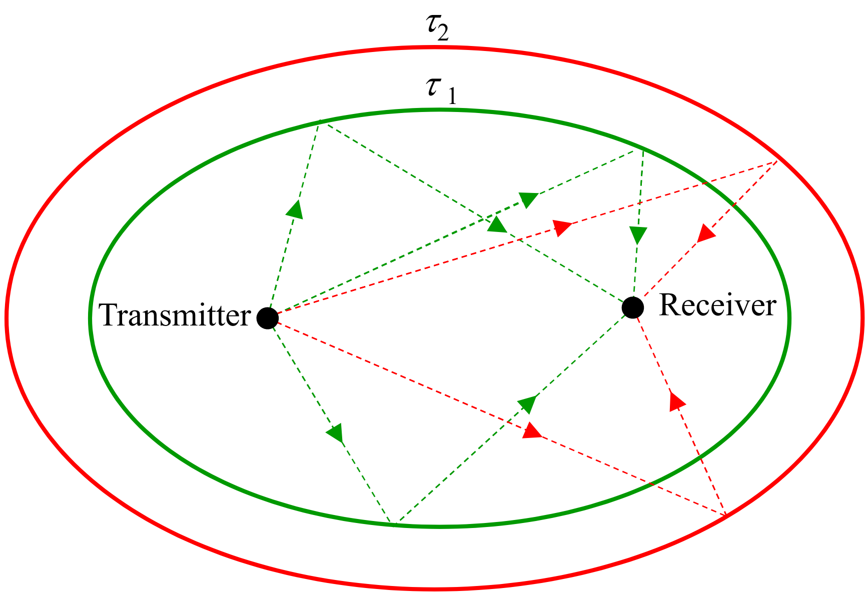
- **Frequency-non-selective:** if the differential of path delays $\tau_i - \tau_j$ are small compared to the duration of a modulated symbol, τ_n are all approximately equal to $\hat{\tau}$

$$g(\tau,t) \cong \sum_{n=1}^{N} \alpha_n(t) e^{-j\phi_n(t)} \delta(\tau - \hat{\tau}) = g(t)\delta(\tau - \hat{\tau})$$



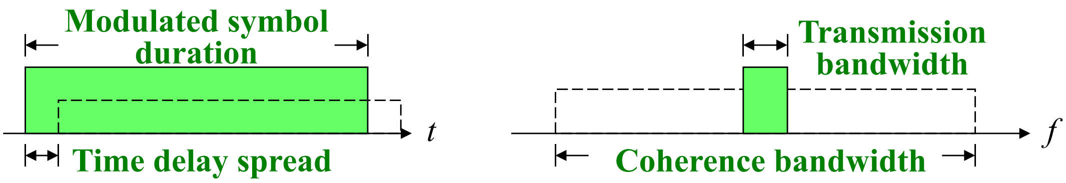
Freq.-Selective & -Non-Selective Fading (Cont.)

- **Frequency-selective:** if the differential of path delays $\tau_i - \tau_j$ are comparable to the duration of a modulated symbol



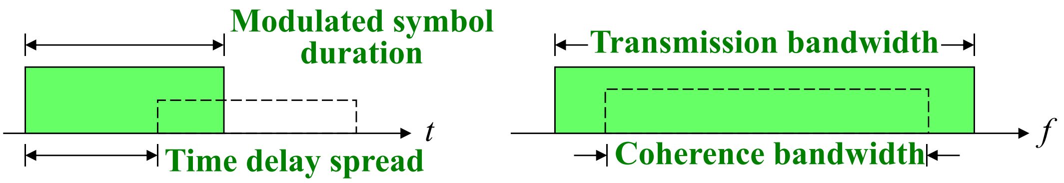
Frequency-Non-Selective Multipath Fading

- **Frequency-non-selective multipath fading:**
  - Narrow-band transmission
  - Signal bandwidth << coherence bandwidth
  - The inverse of the signal bandwidth >> time spread of the propagation path delay
  - Modulated symbol duration >> time spread of the propagation path delay
  - All frequency components experience **the same random attenuation and a linear phase shift**
  - Very little or no distortion ⇒ **no ISI**, do not need equalization



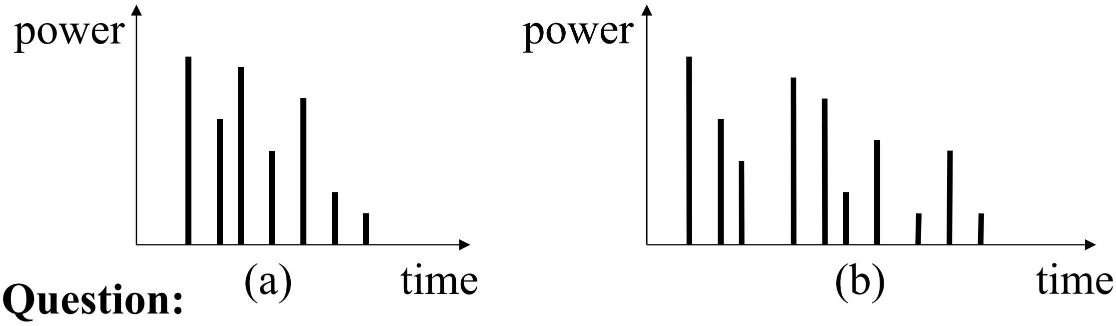
Frequency-Selective Multipath Fading

- **Frequency-selective multipath fading:**
  - Wide-band transmission
  - Signal bandwidth >≈ coherence bandwidth
  - The inverse of the signal bandwidth ≈< the time spread of the propagation path delay
  - Modulated symbol (or chip) duration ≈< time spread of the propagation path delay
  - Different frequency components may experience **different random attenuation and a non-linear phase shift**
  - Significant distortion ⇒ ISI, **equalization or RAKE is need**



Question

- The multipath delay profiles of two propagation channels are shown as follows:



- **Question:**
  - These two channels are **Frequency-selective or Frequency-non-selective**?
  - How to determine the environment is **Frequency-selective or Frequency-non-selective**?
  - Can we say that an environment is **Frequency-selective or Frequency-non-selective** for all communication systems?

Frequency-Non-Selective (Flat) Multipath Fading

Freq.-Non-Selective Multipath Fading

- At any time t, the random phase $\phi_n(t)$ may result in the **constructive** or **destructive** addition of the N components
- If the differential of path delays $\tau_i - \tau_j$ is small compared to the duration of a modulated symbol, for all $i \neq j$, all the path delays are approximately equal to $\hat{\tau}$
- Since the carrier frequency is very high, small differences in the path delays will correspond to **large differences** in $\phi_n(t)$

  $\Rightarrow$ The received signal still experiences fading
- The channel impulse response can be approximated as

$$g(\tau,t) = \sum_{n=1}^{N} \alpha_n(t) e^{-j\phi_n(t)} \delta(\tau - \tau_n(t)) \approx g(t)\delta(\tau - \hat{\tau})$$

- The corresponding channel transfer function is

Impulse response

$$T(t,f) = \mathbb{F}\{g(t,\underline{\tau})\} = \mathbb{F}\{g(t)\delta(\tau - \hat{\tau})\} = \underline{g(t)}e^{-j2\pi f\hat{\tau}}$$

Freq.-Non-Selective Multipath Fading (Cont.)

- The amplitude response is $|T(t,f)| = |g(t)|$
- **All frequency components** in the received signal are subject to the **same complex gain** $g(t)$
  - The phase is linear with respect to $f \Rightarrow$ **constant delay** for all f $\Rightarrow$ **no distortion**
- The received signal is said to exhibit **flat fading**
  - It holds for the corresponding frequency components only, i.e., the frequency components in the transmission bandwidth

Doppler Power Spectrum

Received Signal Correlation

- By assuming the transmission of an **unmodulated carrier**
- The received band-pass signal is

Narrow band signal
Freq.-Non-Selective

$$r(t) = \mathrm{Re}\left\{ \tilde{r}(t) e^{j2\pi f_c t} \right\} \qquad \boxed{\tilde{s}(t) = 1}$$

$$\tilde{r}(t) = \sum_{n=1}^{N} \alpha_n(t)\, e^{-j\phi_n(t)} \tilde{s}(t - \tau_n(t)) = \sum_{n=1}^{N} \alpha_n(t)\, e^{-j\phi_n(t)}$$

$$= g_I(t) + j g_Q(t)$$

– where

$$g_I(t) = \sum_{n=1}^{N} \alpha_n(t) \cos\phi_n(t) \qquad g_Q(t) = -\sum_{n=1}^{N} \alpha_n(t) \sin\phi_n(t)$$

$$\boxed{e^{j\theta} = \cos\theta + j\sin\theta}$$

- The band-pass signal can be expressed as

$$r(t) = g_I(t)\cos 2\pi f_c t - g_Q(t)\sin 2\pi f_c t$$

Received Signal Correlation (Cont.)

- It is assumed that these random processes are all wide sense stationary (WSS)
  - $f_{D,n}(t) = f_{D,n}$, $\alpha_n(t) = \alpha_n$, and $\tau_n(t) = \tau_n$
- The autocorrelation of $r(t)$: (for an arbitrary **time difference τ**)

$$\phi_{rr}(\boldsymbol{\tau}) = E\left[r(t)r(t+\boldsymbol{\tau})\right] \leftarrow \boxed{\text{Auto-correlation time separation}}$$

$$= E\left[g_I(t)g_I(t+\boldsymbol{\tau})\right]\cos 2\pi f_c \boldsymbol{\tau} - E\left[g_Q(t)g_I(t+\boldsymbol{\tau})\right]\sin 2\pi f_c \boldsymbol{\tau}$$

$$= \phi_{g_I g_I}(\boldsymbol{\tau})\cos 2\pi f_c \boldsymbol{\tau} - \phi_{g_Q g_I}(\boldsymbol{\tau})\sin 2\pi f_c \boldsymbol{\tau}$$

$$\boxed{\begin{aligned}\phi_{g_I g_I}(\boldsymbol{\tau}) &= \phi_{g_Q g_Q}(\boldsymbol{\tau});\\ \phi_{g_I g_Q}(\boldsymbol{\tau}) &= -\phi_{g_Q g_I}(\boldsymbol{\tau})\end{aligned}}$$

$$\boxed{\begin{aligned}\sin x \sin y &= \tfrac{1}{2}\left[\cos(x-y) - \cos(x+y)\right]\\ \cos x \cos y &= \tfrac{1}{2}\left[\cos(x-y) + \cos(x+y)\right]\\ \sin x \cos y &= \tfrac{1}{2}\left[\sin(x+y) + \sin(x-y)\right]\\ \cos x \sin y &= \tfrac{1}{2}\left[\sin(x+y) - \sin(x-y)\right]\end{aligned}}$$

Received Signal Correlation (Cont.)

- According to $\phi_n(t) = 2\pi\left[\left(f_c + f_{D,n}(t)\right)\tau_n(t) - f_{D,n}(t)t\right]$, $\tau_n(t) \approx \hat{\tau}$ and $g_I(t) = \sum_{n=1}^{N}\alpha_n(t)\cos\phi_n(t)$, we have

Different paths are uncorrelated for uniform random phase

$$\phi_{g_I g_I}(\boldsymbol{\tau}) = E\left[g_I(t)g_I(t+\boldsymbol{\tau})\right] = \Omega_p \times E_{\hat{\tau},\theta_n}\left[\cos\phi_n(t)\cos\phi_n(t+\boldsymbol{\tau})\right]$$

$$= \frac{\Omega_p}{2}\left\{E_{\hat{\tau},\theta_n}\Big[\underbrace{\overbrace{\cos 2\pi f_{D,n}\boldsymbol{\tau}}^{x-y}}_{\text{A constant for } \hat{\tau}}\Big] + E_{\hat{\tau},\theta_n}\Big[\overbrace{\cos 2\pi[2(f_c + f_{D,n})\hat{\tau} - 2f_{D,n}t - f_{D,n}\boldsymbol{\tau}]}^{x+y}\Big]\right\}$$

$$= \frac{\Omega_p}{2}E_{\theta_n}\left[\cos\left(2\pi f_m \boldsymbol{\tau}\cos\theta_n\right)\right] + 0 \quad \left(\because f_c\hat{\tau} >> 1 \text{ and } f_{D,n}(t) = f_m\cos\theta_n(t)\right)$$

  - where the total received envelope power is

$$\Omega_p = E\left[g_I^2(t)\right] + E\left[g_Q^2(t)\right] = \sum_{n=1}^{N}E\left[\alpha_n^2\right]$$

$\phi_n(t)$ is uniformly distributed over [-π, π]

- Similarly, we have

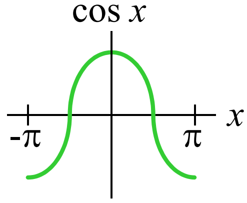
$$\phi_{g_I g_Q}(\boldsymbol{\tau}) = E_{\hat{\tau},\theta_n}\left[g_I(t)g_Q(t+\boldsymbol{\tau})\right] = \frac{\Omega_p}{2}E_{\theta_n}\left[\sin\left(2\pi f_m \boldsymbol{\tau}\cos\theta_n\right)\right]$$

Received Signal Correlation – IS

- For **isotropic scattering (IS)**: θ_n is uniformly distributed over $[-\pi, \pi]$

Even function

$$\phi_{g_I g_I}(\boldsymbol{\tau}) = \frac{\Omega_p}{2}\frac{1}{2\pi}\int_{-\pi}^{\pi}\cos(2\pi f_m \boldsymbol{\tau}\cos\theta)\,d\theta$$



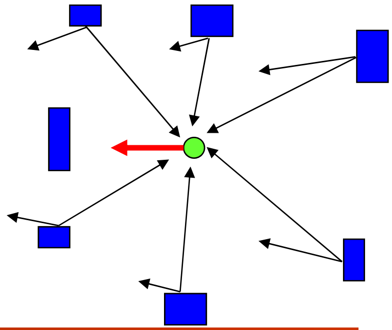
$$= \frac{\Omega_p}{2}\frac{1}{\pi}\int_{0}^{\pi}\cos(2\pi f_m \boldsymbol{\tau}\sin\theta)\,d\theta = \frac{\Omega_p}{2}J_0(2\pi f_m \boldsymbol{\tau})$$

  – $J_0(x)$ is the zero-order Bessel function of the first kind

Odd function

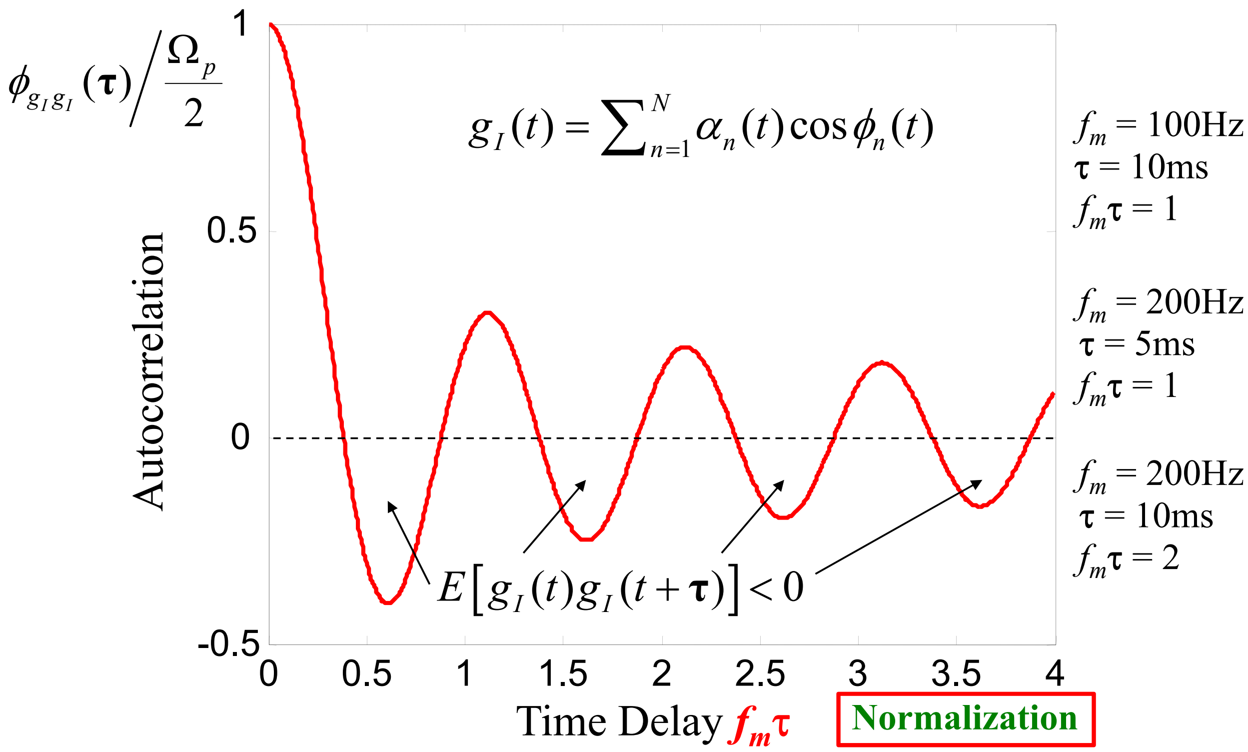
$$\phi_{g_I g_Q}(\boldsymbol{\tau}) = \frac{\Omega_p}{2}\frac{1}{2\pi}\int_{-\pi}^{\pi}\sin(2\pi f_m \boldsymbol{\tau}\cos\theta)\,d\theta = 0$$

$$\boxed{\because \sin(x) = -\sin(-x)}$$



- $g_I(t)$ and $g_Q(t)$ are uncorrelated

Received Signal Correlation – IS (Cont.)



Received Signal Spectrum – IS (Cont.)

- The power spectral density of $g_I(t)$ is

$$S_{g_I g_I}(f) = \mathbb{F}\left\{\phi_{g_I g_I}(\boldsymbol{\tau})\right\} = \begin{cases} \dfrac{\Omega_p}{2\pi f_m} \dfrac{1}{\sqrt{1-\left(f/f_m\right)^2}} & |f| \le f_m \\ 0 & \text{otherwise} \end{cases}$$

- The received complex envelope of $r(t)$ is

$$\tilde{r}(t) = g(t) = g_I(t) + jg_Q(t)$$

$$\Rightarrow \phi_{gg}(\boldsymbol{\tau}) = \frac{1}{2}E\left[g^*(t)g(t+\boldsymbol{\tau})\right] = \phi_{g_I g_I}(\boldsymbol{\tau}) + j\phi_{g_I g_Q}(\boldsymbol{\tau})$$

- The power spectral density of $g(t)$ (**Doppler power spectrum**) is

$$S_{gg}(f) = S_{g_I g_I}(f) + jS_{g_I g_Q}(f)$$

Received Signal Spectrum – IS (Cont.)

- For the received band-pass signal $r(t)$, we have

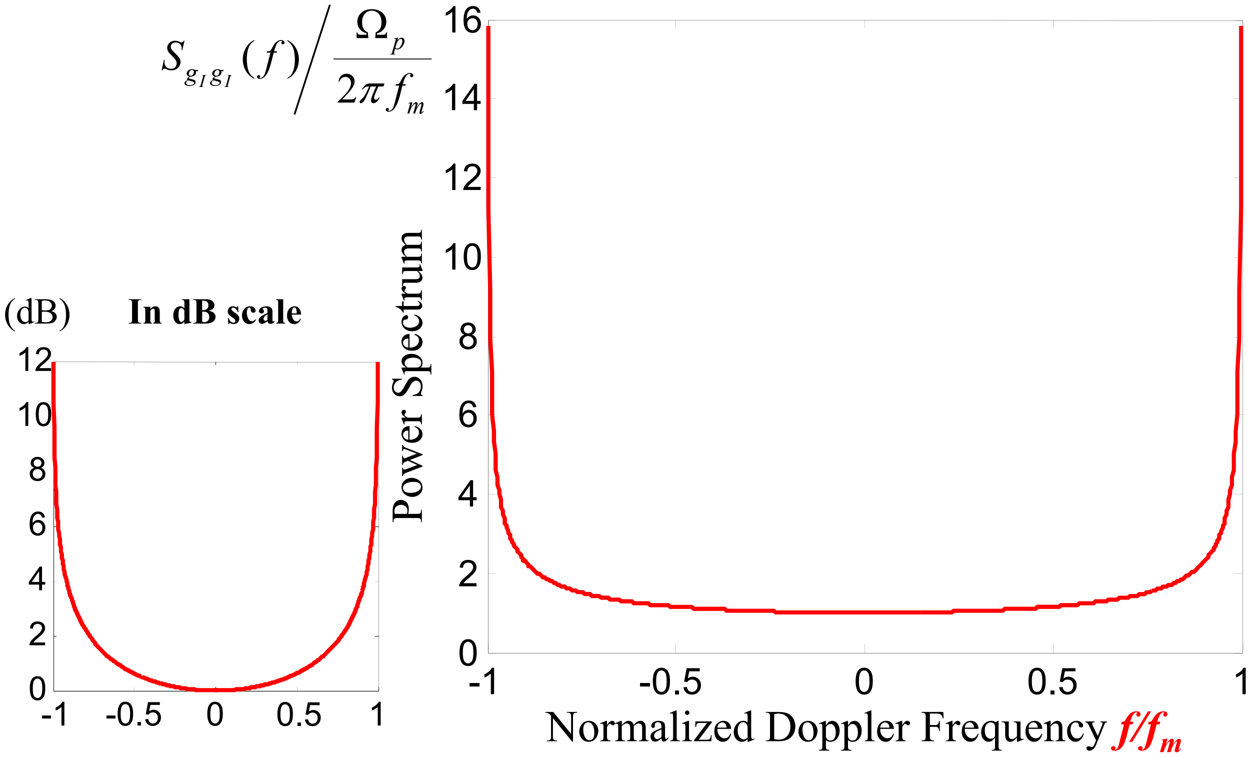
$$\phi_{rr}(\boldsymbol{\tau}) = \Re\left[\phi_{gg}(\boldsymbol{\tau})e^{j2\pi f_c \tau}\right]$$

- Since $\phi_{g_I g_Q}(\boldsymbol{\tau}) = 0$, we have the PSD of $r(t)$ as

$$\begin{aligned} S_{rr}(f) &= \frac{1}{2}\left[S_{gg}(f-f_c) + S_{gg}(-f-f_c)\right] \\ &= \frac{1}{2}\left[S_{g_I g_I}(f-f_c) + S_{g_I g_I}(-f-f_c)\right] \\ &= \frac{\Omega_p}{4\pi f_m}\frac{1}{\sqrt{1-\left(|f-f_c|/f_m\right)^2}}, \qquad |f-f_c| \le f_m \end{aligned}$$

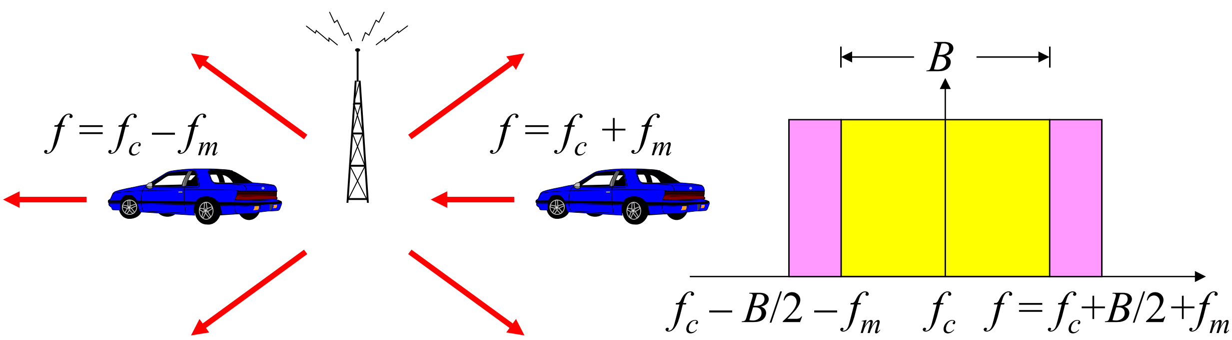
- $S_{rr}(t)$ is limited to $|f-f_c| \le f_m$

Received Signal Spectrum – IS (Cont.)



Question

- **Question:**
  - Why the power spectral density of an unmodulated carrier is limited to $|f - f_c| \le f_m$?
  - For a transmission with the transmission bandwidth B, what is the frequency range of the received power spectrum?

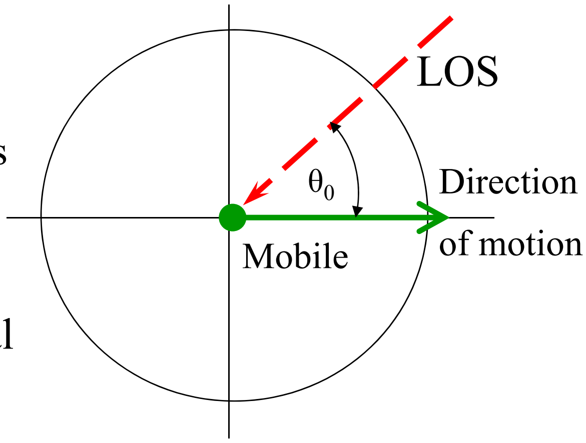


Received Signal Spectrum with LOS – IS

- If an **LOS** or a **strong specular component** is present in the received signal and arrives at angle θ_0: **Ricean／Rician fading**

$$S_{gg}(f)=\begin{cases}\dfrac{1}{K+1}\dfrac{\Omega_p}{2\pi f_m}\dfrac{1}{\sqrt{1-(f/f_m)^2}}+\dfrac{K}{K+1}\dfrac{\Omega_p}{2}\delta(f-f_m\cos\theta_0),\ 0\le|f|\le f_m\\ 0, \qquad \text{otherwise}\end{cases}$$

  - where K is the **Rice factor**: the ratio of the power in the **specular** and **scatter** components of the received signal



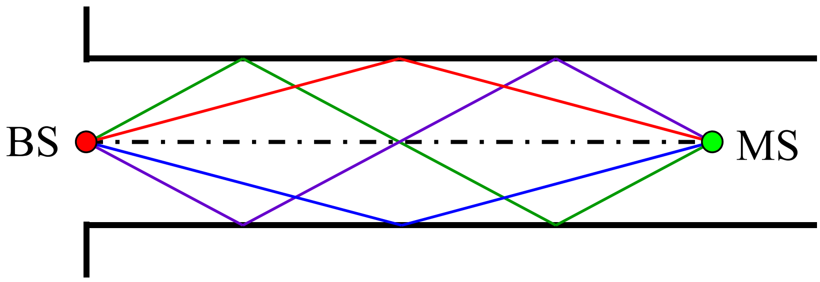
- The PSD is the same as Fig. 2.4, except for an additional **discrete tone** at $f_c+f_m\cos\theta_0$

Received Signal Correlation (Microcellular)

- In microcellular environment, the plane waves may be **channeled by the buildings** along the streets and arrive at the receiver from just **one direction**
  - The scattering is **non-isotropic**

$$p(\theta)=\begin{cases}\dfrac{\pi}{4|\theta_m|}\cos\left(\dfrac{\pi}{2}\times\dfrac{\theta}{\theta_m}\right), & |\theta|\le|\theta_m|\le\pi/2\\ 0, & \text{elsewhere}\end{cases}$$

  - θ_m: the directivity of the incoming waves



Received Signal Correlation (Microcell) (Cont.)

- According to

$$\phi_{g_I g_I}(\boldsymbol{\tau}) = \frac{\Omega_p}{2} E_{\theta_n}\left[\cos\left(2\pi f_m \boldsymbol{\tau} \cos\theta_n\right)\right]$$

$$\phi_{g_I g_Q}(\boldsymbol{\tau}) = \frac{\Omega_p}{2} E_{\theta_n}\left[\sin\left(2\pi f_m \boldsymbol{\tau} \cos\theta_n\right)\right]$$

- We have

$$\phi_{g_I g_I}(\boldsymbol{\tau}) = \frac{\Omega_p}{2}\int_{-\pi}^{\pi} \cos(2\pi f_m \boldsymbol{\tau}\cos\theta)\underline{p(\theta)}\, d\theta \neq \frac{\Omega_p}{2} J_0(2\pi f_m \boldsymbol{\tau})$$

$$\phi_{g_I g_Q}(\boldsymbol{\tau}) = \frac{\Omega_p}{2}\int_{-\pi}^{\pi} \sin(2\pi f_m \boldsymbol{\tau}\cos\theta)\underline{p(\theta)}\, d\theta \neq 0$$

Fading Characteristics (Received Envelope/Power Distribution)

Rayleigh Fading

- **Rayleigh Fading:** the received complex low-pass signal is modeled as a complex Gaussian random process
  - $g_I(t)$ and $g_Q(t)$ are independent **zero-mean** Gaussian RVs
  - The received **complex envelope** $\alpha(t) = |\, g(t)\,|$ has a Rayleigh distribution

$$p_\alpha(x) = \frac{x}{\sigma^2}\exp\left[-\frac{x^2}{2\sigma^2}\right], \quad x \ge 0$$

$$\boxed{\begin{aligned} g_I(t) &= \sum_{n=1}^{N} \alpha_n(t)\cos\phi_n(t) \\ g_Q(t) &= -\sum_{n=1}^{N} \alpha_n(t)\sin\phi_n(t) \end{aligned}}$$

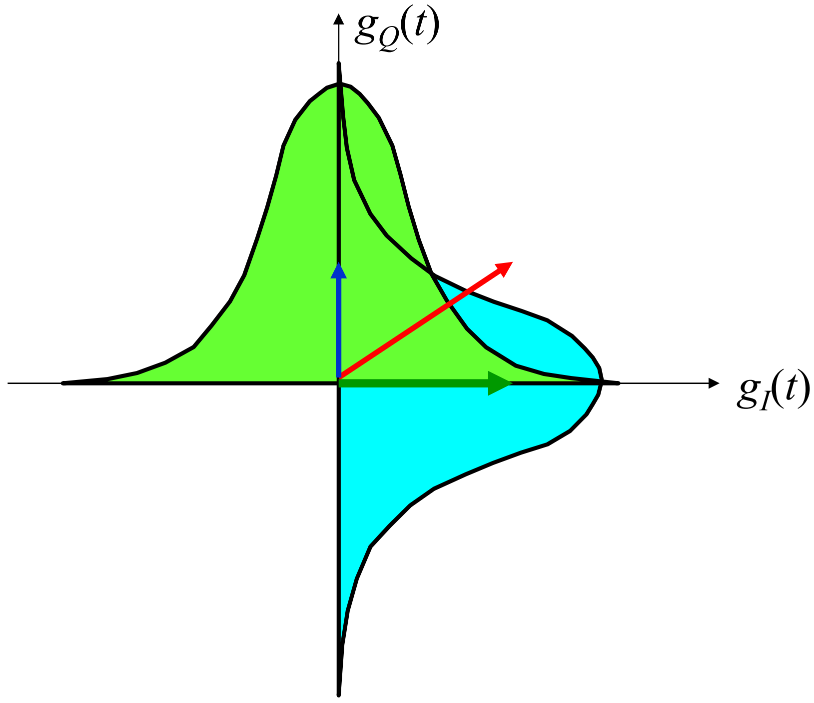
- The average power is

$$E[\alpha^2] = \Omega_p = 2\sigma^2$$

- The squared-envelope (power) $\alpha^2(t) = |\, g(t)\,|^2$ has an exponential distribution

$$p_{\alpha^2}(x) = \frac{1}{\Omega_p}\exp\left[-\frac{x}{\Omega_p}\right], \quad x \ge 0$$

Rayleigh Fading (Cont.)



Ricean Fading

- **Ricean Fading:** the received complex low-pass signal contains a LOS or a strong specular component
  - $g_I(t)$ and $g_Q(t)$ are independent Gaussian RVs with **non-zero mean** $m_I(t)$ and $m_Q(t)$ ($m_I(t)$ and $m_Q(t)$ depend on the LOS and θ_0)
  - The complex envelope $\alpha(t) = |\,g(t)\,|$ has a Ricean distribution

$$p_\alpha(x) = \frac{x}{\sigma^2}\exp\left[-\frac{x^2+s^2}{2\sigma^2}\right] I_0\left(\frac{xs}{\sigma^2}\right) \quad x \ge 0$$

Power of the LOS component

$$s^2 = m_I^2(t) + m_Q^2(t), \qquad K = s^2 / 2\sigma^2$$

  - The modified Bessel function of the first kind of zero order

$$I_0(x) = \int_0^{2\pi} \exp(x\cos\psi)\, d\psi \Big/ 2\pi$$

  - Rice factor $K = 0$: Rayleigh fading
  - Rice factor $K = \infty$: the channel does not exhibit fading

Ricean Fading (Cont.)

- The average power is

$$E[\alpha^2] = \Omega_p = s^2 + 2\sigma^2$$

$$s^2 = \frac{K\Omega_p}{K+1}, \qquad 2\sigma^2 = \frac{\Omega_p}{K+1}$$

- The squared-envelope $\alpha^2(t) = |\,g(t)\,|^2$ has a non-central chi-square distribution

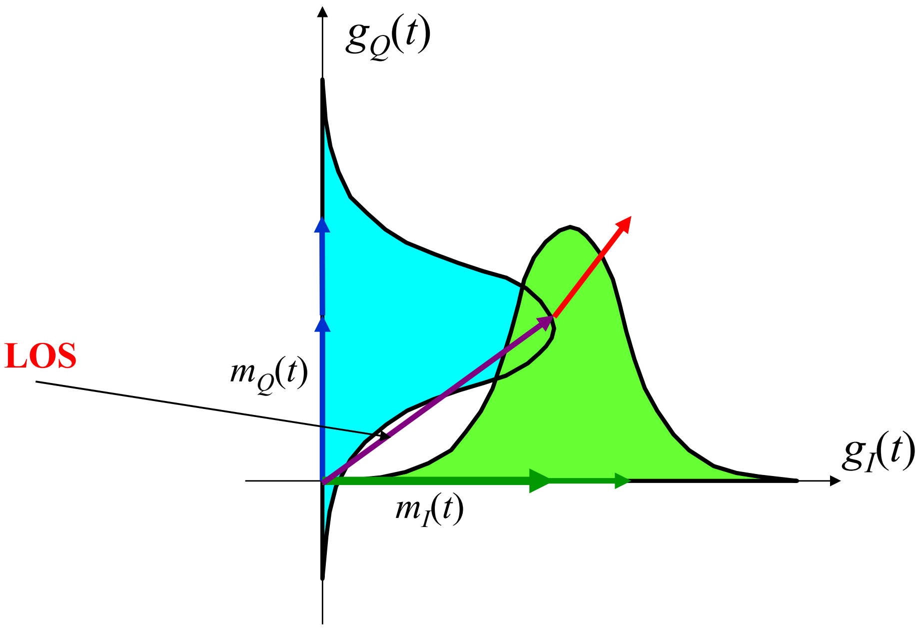
$$p_{\alpha^2}(x) = \frac{(K+1)}{\Omega_p}\exp\left[-K - \frac{(K+1)x}{\Omega_p}\right] I_0\left(2\sqrt{\frac{K(K+1)x}{\Omega_p}}\right), \quad x \ge 0$$

- The phase is not uniformly distributed over $[-\pi, \pi]$ for $K \ne 0$

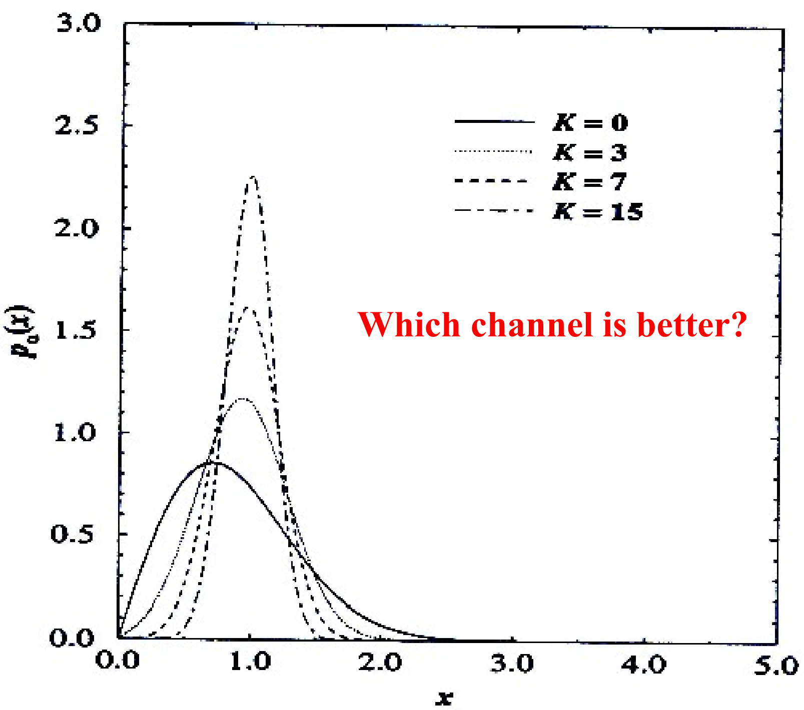
$$\phi(t) = \tan^{-1}\left(x_Q(t)/x_I(t)\right)$$

  - For $K = 0$: Rayleigh fading $p_\phi(x) = 1/2\pi, \quad -\pi \le x \le \pi$

Ricean Fading (Cont.)



Rayleigh and Ricean Distributions



Nakagami Fading

- **Nakagami Fading:** provides a closer match to some experimental data
  - The received complex envelope $\alpha(t) = |g(t)|$ has a Nakagami distribution

$$p_\alpha(x) = \frac{2m^m x^{2m-1}}{\Gamma(m)\Omega_p^m} \exp\left[-\frac{mx^2}{\Omega_p}\right], \quad m \geq \frac{1}{2}$$

    - where $\Gamma(m)$ is the Gamma function defined as

$$\Gamma(m) = \int_0^\infty u^{m-1} e^{-u}\, du$$
$$= (m-1)!, \quad \text{if } m \text{ is a positive integer}$$

  - $m = 1$: Rayleigh fading
  - $m = 1/2$: one-sided Gaussian
  - $m = \infty$: no fading

Nakagami Fading (Cont.)

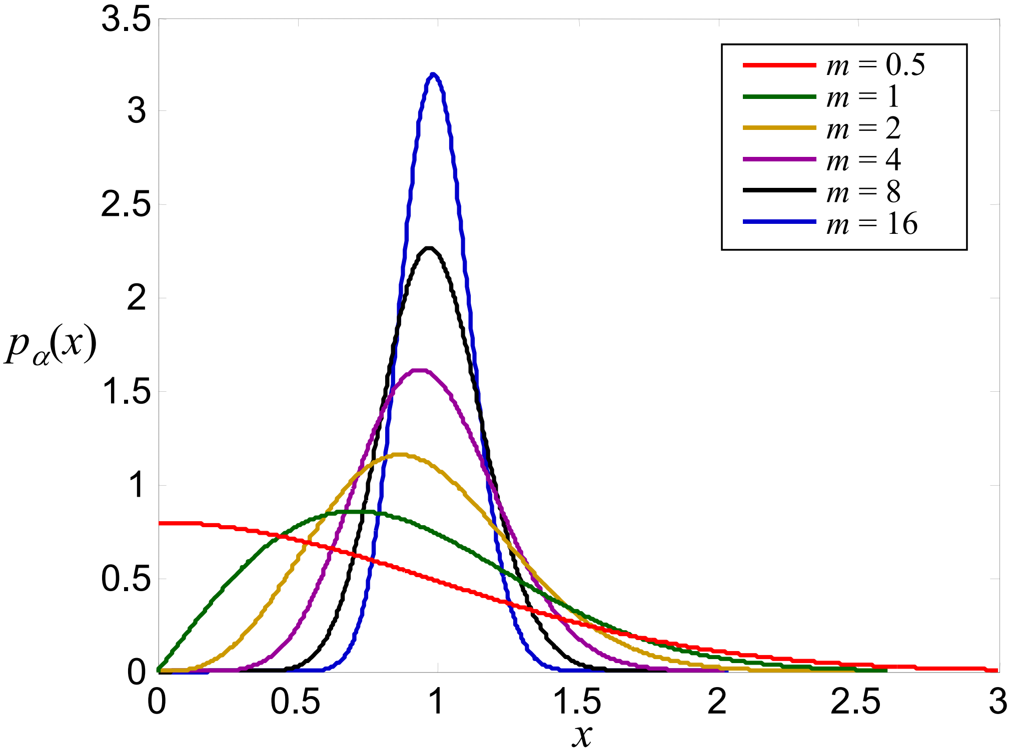
- Nakagami distribution can model fading conditions that are **either more or less** severe than Rayleigh fading
- Ricean fading can be closely approximated by

$$K = \frac{\sqrt{m^2 - m}}{m - \sqrt{m^2 - m}} \qquad m \geq 1; \qquad m = \frac{(K+1)^2}{(2K+1)}$$

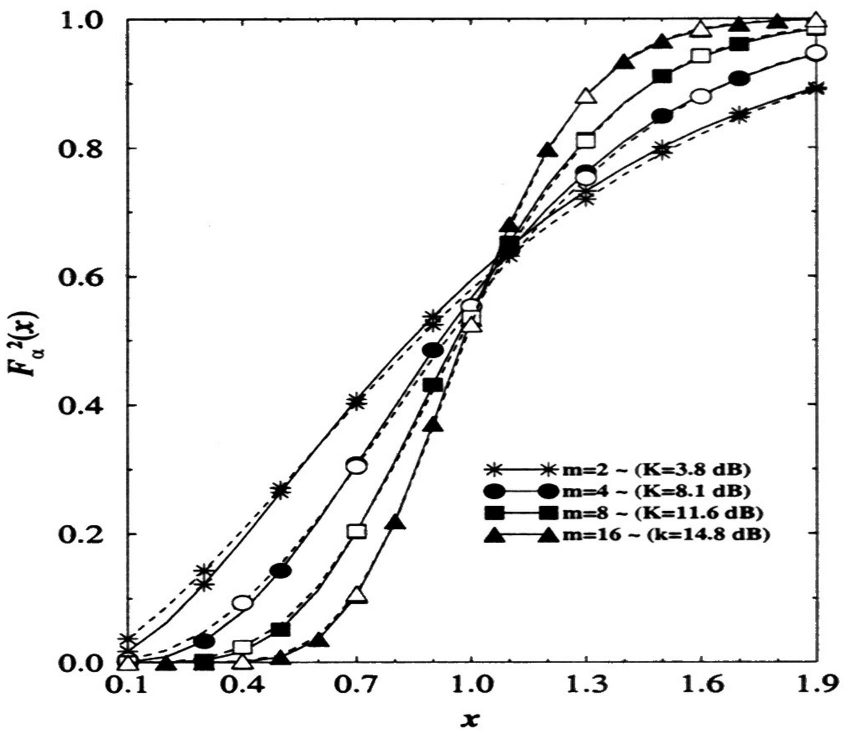
- The Nakagami distribution often leads to closed form analytical expressions
- The squared-envelope $\alpha^2(t) = |g(t)|^2$ has a Gamma density

$$p_{\alpha^2}(x) = \left(\frac{m}{\Omega_p}\right)^m \frac{x^{m-1}}{\Gamma(m)} \exp\left[-\frac{mx}{\Omega_p}\right]$$

Nakagami Fading (Cont.)



Nakagami and Ricean Distributions



Envelope Correlation

- The autocorrelation of the envelope $\alpha(t) = |\, g(t)\,|$:

$$\phi_{\alpha\alpha}(\tau) = E\left[\alpha(t)\alpha(t+\tau)\right] = \frac{\pi}{2}\left|\phi_{gg}(0)\right| F\left[-\frac{1}{2}, -\frac{1}{2}; 1, \frac{\left|\phi_{gg}(\tau)\right|^2}{\left|\phi_{gg}(0)\right|^2}\right]$$

  – where $\left|\phi_{gg}(\tau)\right|^2 = \phi_{g_I g_I}^2(\tau) + \phi_{g_I g_Q}^2(\tau)$

$$= \phi_{g_I g_I}^2(\tau) \quad \text{(isotropic scattering)}$$

$$F\left[-\tfrac{1}{2}, -\tfrac{1}{2}; 1, x\right] = 1 + \tfrac{1}{4}x + \tfrac{1}{64}x^2 + \cdots \text{(Hypergeometric Function)}$$

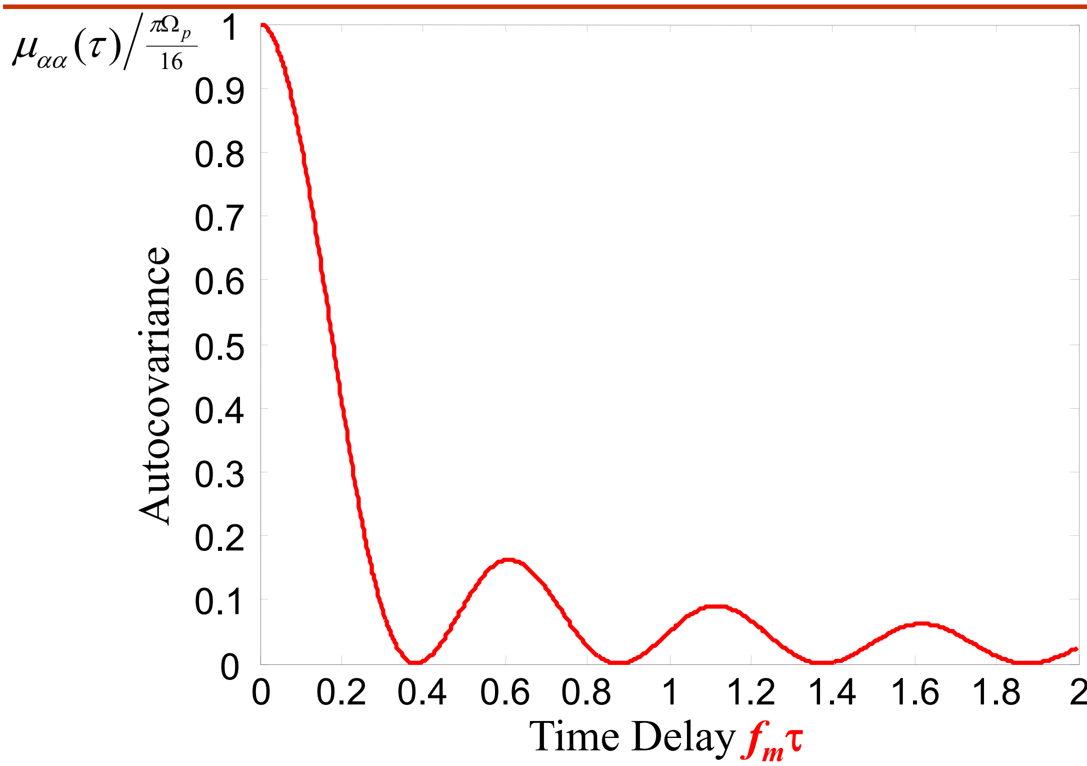
$$\phi_{\alpha\alpha}(\tau) \approx \frac{\pi}{2}\left|\phi_{gg}(0)\right|\left[1 + \frac{1}{4}\frac{\left|\phi_{gg}(\tau)\right|^2}{\left|\phi_{gg}(0)\right|^2}\right]$$

- The autocovariance function:

$$\mu_{\alpha\alpha}(\tau) = E\left[\alpha(t)\alpha(t+\tau)\right] - E\left[\alpha(t)\right]E\left[\alpha(t+\tau)\right]$$

$$= \tfrac{\pi}{8\left|\phi_{gg}(0)\right|}\left|\phi_{gg}(\tau)\right|^2 = \tfrac{\pi\Omega_p}{16} J_0^2\left(2\pi f_m \tau\right)$$

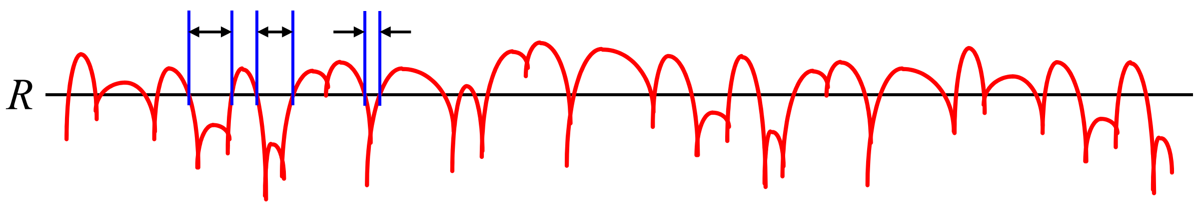
Envelope Correlation (Cont.)



Envelope Level Crossing Rate and Average Envelope Fade Duration

The Impact of Multipath Fading

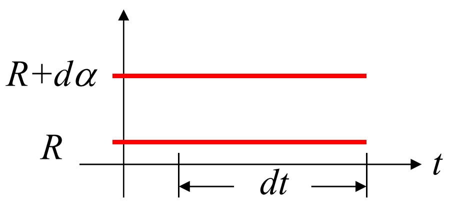
- The receive performance is severely degraded in a **deep fade region**.
  - For example, the received signal level is below a threshold *R*
- We care the following two things:
  - How often will deep fading occur?
    - **Envelope Level Crossing Rate**
  - How long will the deep fading last?
    - **Average Envelope Fade Duration**



Envelope Level Crossing Rate

- L_R: the rate at which the envelope crosses level R in the positive (or negative) going direction
- $\dot{\alpha}$: the envelope slope
  - $\dot{\alpha}$ is either **positive** (for positive going direction) or **negative** (for negative going direction)
- $p(\alpha, \dot{\alpha})$: the join pdf of α and $\dot{\alpha}$
- dt: the observation time interval
- For given values of $\alpha = R$ and $\dot{\alpha}$, the probability is

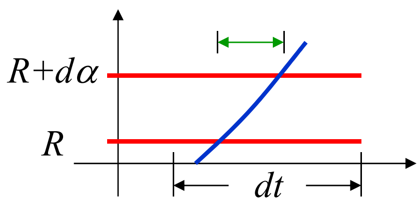
$$p(R,\dot{\alpha})\, d\alpha\, d\dot{\alpha}$$

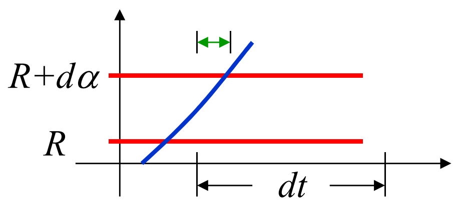


Envelope Level Crossing Rate (Cont.)

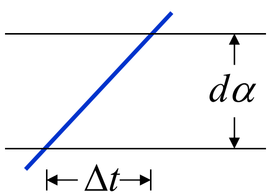
- The expected amount of time spent in the interval $(R, R + d\alpha)$ for given values of $\dot{\alpha}$ and dt is

$$p(R,\dot{\alpha})\, d\alpha\, d\dot{\alpha}\, dt$$





- The time required to cross the interval $d\alpha$ once for a given $\dot{\alpha}$ is $d\alpha/\dot{\alpha}$
  - The time spent in $(R, R + d\alpha)$ for one positive going direction cross



$$\dot{\alpha} = d\alpha/\Delta t$$

Envelope Level Crossing Rate (Cont.)

- The expected number of crossings of the envelope α within the interval $(R, R+d\alpha)$ for a given $\dot{\alpha}$ is

$$\left(p(R,\dot{\alpha})\,d\alpha\,d\dot{\alpha}\,dt\right)/\left(d\alpha/\dot{\alpha}\right)=\dot{\alpha}\;p(R,\dot{\alpha})\,d\dot{\alpha}\,dt$$

- The expected number of crossings in a time interval T for a given $\dot{\alpha}$ is

$$\int_0^T \dot{\alpha}\;p(R,\dot{\alpha})\,d\dot{\alpha}\,dt=\dot{\alpha}\;p(R,\dot{\alpha})\,d\dot{\alpha}\,T$$

- The excepted number of positive going direction crossings:

$$N_R=T\int_0^\infty \dot{\alpha}\;p(R,\dot{\alpha})\,d\dot{\alpha}$$

All slopes are counted.

- The envelope level crossing rate:

$$L_R=\int_0^\infty \dot{\alpha}\;p(R,\dot{\alpha})\,d\dot{\alpha}$$

Envelope Level Crossing Rate – Ricean

- For Ricean fading:

$$p(\alpha,\dot{\alpha})=\sqrt{\frac{1}{2\pi b_2}}\exp\left\{-\frac{\dot{\alpha}^2}{2b_2}\right\}\times\frac{\alpha}{b_0}\exp\left\{-\frac{(\alpha^2+s^2)}{2b_0}\right\}I_0\left(\frac{\alpha s}{b_0}\right)=p(\dot{\alpha})p(\alpha)$$

  – where $b_2 = b_0(2\pi f_m)^2/2$ and $2b_0$ is the power of the scatter component of the received signal

- The envelope level crossing rate is

$$L_R=\sqrt{2\pi(K+1)}f_m\rho e^{-K-(K+1)\rho^2}I_0\left(2\rho\sqrt{K(K+1)}\right)$$

  – where $\rho=\dfrac{R}{\sqrt{\Omega_p}}=\dfrac{R}{R_{rms}}$

  – $\sqrt{\Omega_p}\triangleq R_{rms}$: the rms envelope level

Envelope Level Crossing Rate – Rayleigh

- For Rayleigh fading ($K = 0$):

$$L_R = \sqrt{2\pi} f_m \rho e^{-\rho^2}$$

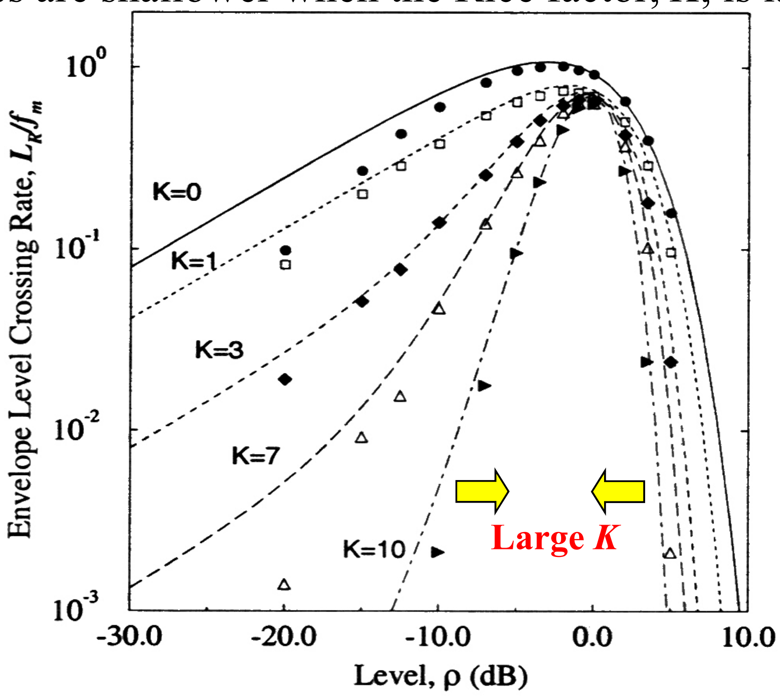
- Maximum LCR: around $\boldsymbol{\rho = 0}$ **dB** (nearly independent of K)
  – For Rayleigh fading channel:

$$\frac{dL_R}{d\rho} = \sqrt{2\pi} f_m \left(e^{-\rho^2} - 2\rho^2 e^{-\rho^2}\right) = \sqrt{2\pi} f_m \left(1 - 2\rho^2\right) e^{-\rho^2} = 0$$

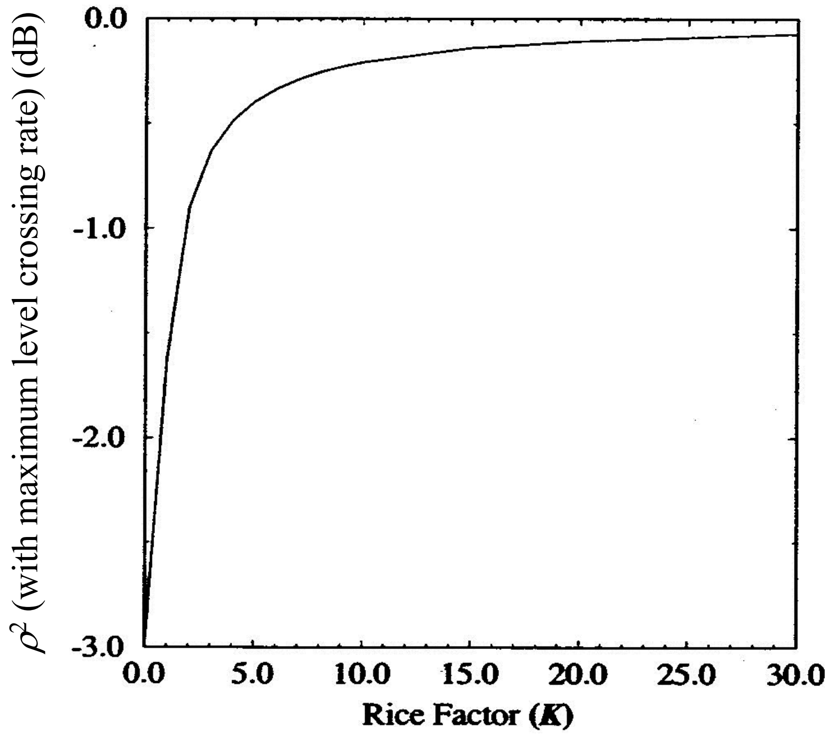
$$\Rightarrow \rho = 1/\sqrt{2}$$

Envelope Level Crossing Rate (Cont.)

- The fades are shallower when the Rice factor, K, is larger



Envelope Level Crossing Rate (Cont.)

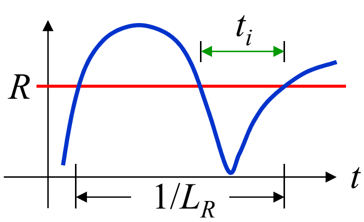


Average Envelope Fade Duration

- Consider a very long observation time interval T
- The probability that the received envelope level is less than R can be expressed as

Can be obtained based on the envelope distribution, i.e., Rayleigh/Ricean distribution

$$P(\alpha \le R) = \frac{1}{T}\sum_i t_i$$



- The average envelope **fade duration** is

Total number of level crossings

$$\bar{t} = \frac{1}{TL_R}\sum_i t_i = \frac{P(\alpha \le R)}{L_R}$$

  - $1/L_R$ is the mean time interval between two adjacent level-crossings
  - $P(\alpha \le R)$: the probability of $\alpha \le R$
  - Only one interval less than R in the $1/L_R$ duration

Average Envelope Fade Duration (Cont.)

- **Ricean:** $P(\alpha \le R) = \int_0^R p(\alpha)d\alpha = 1 - Q\left(\sqrt{2K}, \sqrt{2(K+1)\rho^2}\right)$
  - where $Q(a,b)$ is the Marcum Q function

$$Q(a,b) \triangleq \int_b^{\infty} \alpha \exp\left[-\tfrac{1}{2}\left(\alpha^2 + a^2\right)\right] I_0(a\alpha)\, d\alpha$$

- The average envelope fade duration is

$$\bar{t} = \frac{1 - Q\left(\sqrt{2K}, \sqrt{2(K+1)\rho^2}\right)}{\sqrt{2\pi(K+1)}\, f_m \rho e^{-K-(K+1)\rho^2} I_0(2\rho\sqrt{K(K+1)})}$$

- **Rayleigh:** $P(\alpha \le R) = \int_0^R p(\alpha)\, d\alpha = 1 - e^{-\rho^2}$
- The average envelope fade duration is

$$\bar{t} = \frac{e^{\rho^2} - 1}{\rho f_m \sqrt{2\pi}}, \qquad L_R = \sqrt{2\pi} f_m \rho e^{-\rho^2}$$

Average Envelope Fade Duration (Cont.)

- The average level crossing rate, zero crossing rate and average fade duration all depend on the velocity of MS
  - $f_m = v/\lambda_c$ and 1 mile = 1.609 km
  - Example:

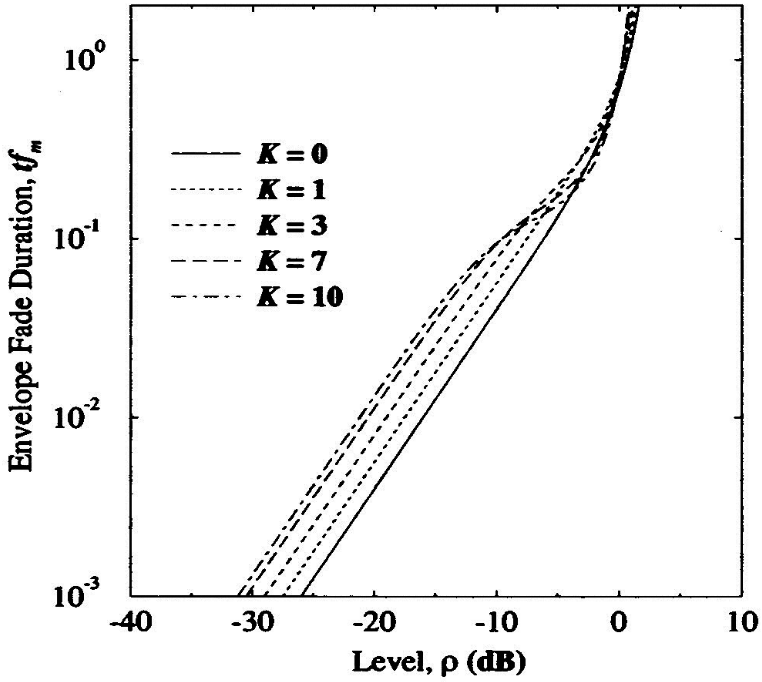
$$v = 60 \text{ mile/hr} = 97 \text{ km/hr} = 27 \text{ m/sec};$$
$$f_c = 900 \text{ MHz} \Rightarrow f_m = 81 \text{ Hz}$$

  - Rayleigh:

$$L_R = 74 \text{ fades/sec at } \rho = 0 \text{ dB}; \quad \bar{t} = 8.5 \text{ ms}$$
$$L_R = 2.0 \text{ fades/sec at } \rho = -20 \text{ dB}; \quad \bar{t} = 50\ \mu\text{s}$$

Average Envelope Fade Duration (Cont.)

- The average fade duration tends to be larger with the Rice factor K

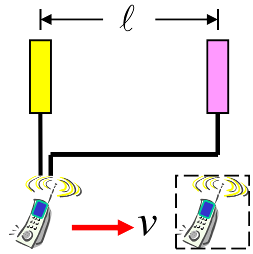


Spatial Correlation

Spatial Correlation

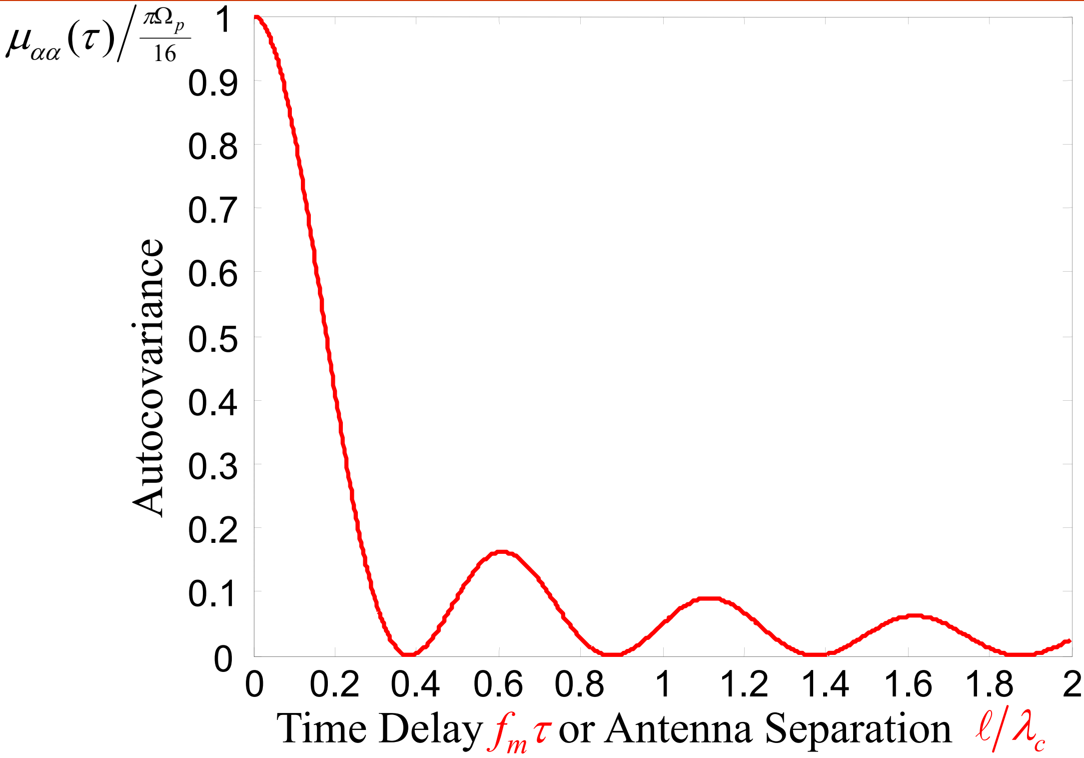
- **Diversity reception:** use two separate receiving antennas to provide uncorrelated diversity branches
- The antenna separation: ℓ
  - By distance-time transformation

$$\ell = v\tau, \quad \ell/\lambda_c = v\tau/\lambda_c = f_m\tau$$



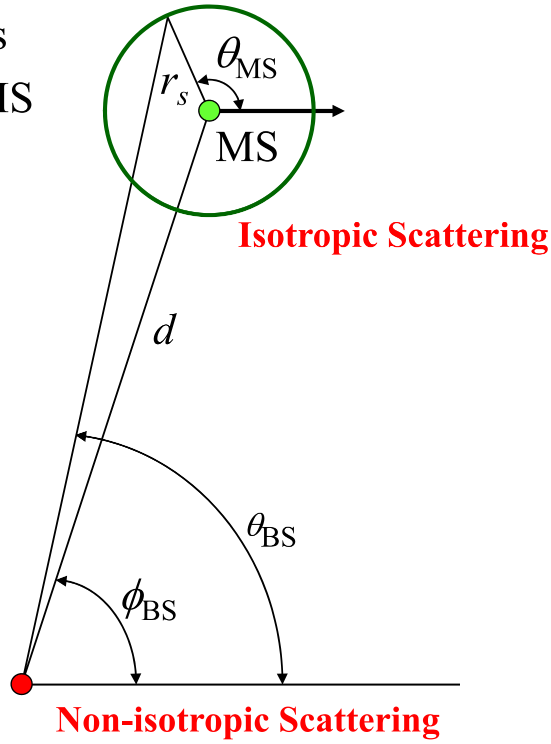
- For the case of **isotropic scattering**:
  - Autocorrelation: $\phi_{g_I g_I}(\ell) = \frac{\Omega_p}{2} J_0(2\pi \ell/\lambda_c)$
  - Autocovariance: $\mu_{\alpha\alpha}(\ell) = \frac{\pi\Omega_p}{16} J_0^2(2\pi \ell/\lambda_c)$
- The normalized envelope autocovariance is zero at $\ell = 0.38\lambda_c$
  - Less than 0.3 for $\ell > 0.38\lambda_c$

Spatial Correlation (Cont.)



Spatial Correlation (Cont.)

- r_s: the radius of primary scatterers
- d: the distance between BS and MS
- ϕ_{BS}: arriving angle

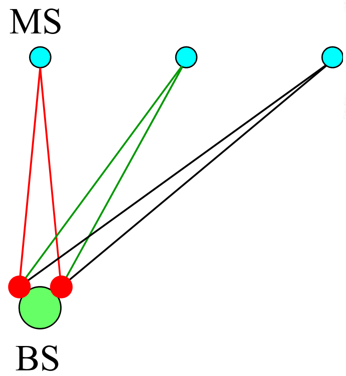


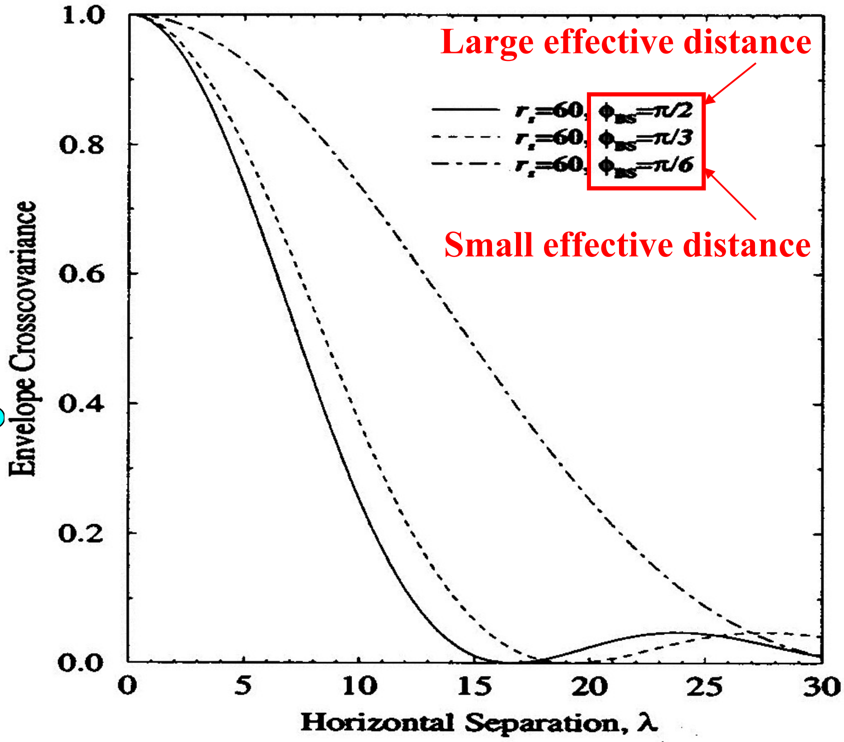
Spatial Correlation (Cont.)

- For an MS (isotropic scattering), the antenna elements should space about **a half-wavelength apart**
- For a BS, the antenna elements separate about $\mathbf{20\lambda_c}$ to obtain a correlation of about 0.7
  - The location of BS antennas is highly above the buildings
  - The arriving plane waves at the BS tend to be concentrated in **a narrow angle** of arrival (non-isotropic scattering)
  - The two antennas located at the BS will view the MS from only a slightly different angle
  - The spatial correlation is higher than isotropic scattering
- Another scheme of diversity reception: **polarization reception**

Spatial Correlation for BS

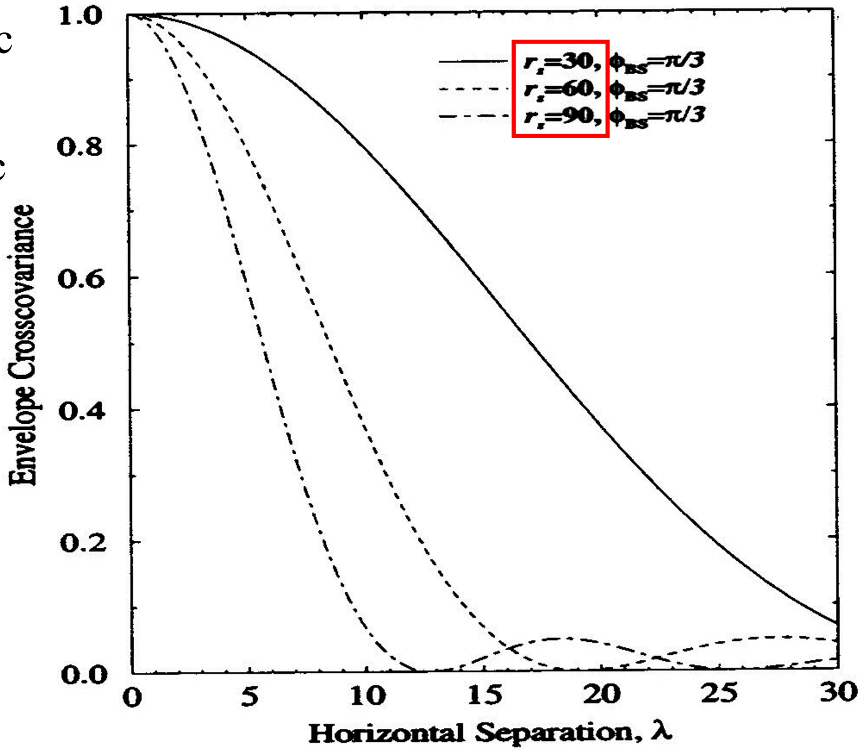
- Almost like a single incoming wave





Spatial Correlation for BS (Cont.)

- $r_s \uparrow \Rightarrow$ isotropic scattering
- $d \downarrow \Rightarrow$ isotropic scattering



Question

- For modern wireless cellular systems, the allocated frequency bands are around 900 MHz and 2 GHz.
- **Question:**
  - Is it possible to implement the spatial diversity reception in an MS? Why?
  - Is it possible to implement the spatial diversity reception in a BS? Why?

Frequency-Selective Multipath Fading

Transmission Functions

- The multipath fading channels can be modeled as **time-variant linear filters**

$\Rightarrow$ Four transmission functions are used for representation
  - Input delay-spread function $g(\tau, t)$
  - Output Doppler-spread function $H(f, \nu)$
  - Time-variant transfer function $T(f, t)$
  - Delay Doppler-spread function $S(\tau, \nu)$
- The parameters:
  - t: time domain
  - f: frequency domain
  - τ: time delay
  - ν: Doppler frequency shift

Transmission Functions (Cont.)

- The time delay (delay spread) determines the channel frequency response
  - The time delay τ can be viewed as the impulse response of the filter $\Rightarrow$ corresponding to the frequency response of the filter
  - The distributions of τ and f vary with time t
  - τ relates to f in different domains
- The varying of time corresponds to the change in the scattering environment (the change of Doppler frequency shift)
  - t relates to ν in different domains
- $t \leftrightarrow \nu$
- $\tau \leftrightarrow f$

Transmission Functions (Cont.)

$$g(\tau, t) \underset{\tau \leftrightarrow f}{\overset{Fourier}{\Leftrightarrow}} T(f, t)$$

$$T(f, t) \underset{t \leftrightarrow v}{\overset{Fourier}{\Leftrightarrow}} H(f, v)$$

$$S(\tau, v) \underset{\tau \leftrightarrow f}{\overset{Fourier}{\Leftrightarrow}} H(f, v)$$

$$g(\tau, t) \underset{t \leftrightarrow v}{\overset{Fourier}{\Leftrightarrow}} S(\tau, v)$$

Classification of Channels

- Three channel types:
  - Wide Sense Stationary (WSS) channel
  - Uncorrelated Scattering (US) channel
  - Wide Sense Stationary Uncorrelated Scattering (WSSUS) channel

Wide Sense Stationary (WSS) Channel

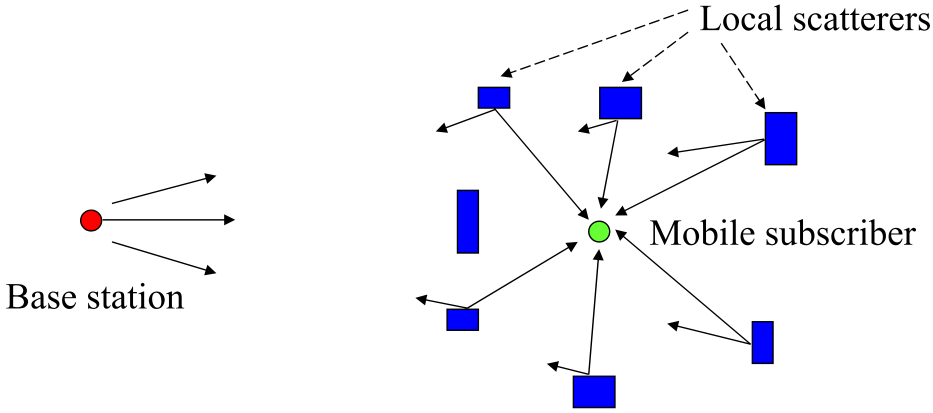
- The fading statistics **remain constant** over short periods of time
- The channel correlation functions depend on the **time difference** Δt
- $t \leftrightarrow v$: the fading characteristics are constant in the time domain $\leftrightarrow$ a delta function in the **correlation** of Doppler frequency shift
  - WSS channels give rise to scattering with **uncorrelated Doppler shifts**
  - The attenuations and phase shifts, associated with signal components **having different Doppler shifts**, are uncorrelated
- **The fading statistics remain constant**
- **Signal components having different Doppler shifts are uncorrelated**

Uncorrelated Scattering (US) Channel

- The attenuations and phase shifts, associated with the paths of **different delays**, are uncorrelated
- $\tau \leftrightarrow f$: the fading characteristics are uncorrelated (delta function) in the delay time domain $\leftrightarrow$ **constant** characteristics in the frequency domain
  - WSS in the frequency variable
  - The correlation functions depend on the frequency difference Δf
- **WSS in the frequency variable**
- **Signal components having different delays are uncorrelated**

WSSUS Channel

- Wide Sense Stationary Uncorrelated Scattering Channel
- The channel displays uncorrelated scattering in both the **time-delay** and **Doppler shift**
- Most of the radio channels can be modeled as WSSUS channels



Multipath Intensity Profile

- For WSSUS, the autocorrelation function of $g(\tau, t)$: $\phi_g(\Delta t; \tau)$
- Multipath intensity profile: For $\Delta t = 0$, $\phi_g(0;\tau) = \phi_g(\tau)$ shows the power profile
  - The average power at channel output of time delay τ
  - It can be viewed as the scattering function averaged over all Doppler shifts
- Average delay: $$\mu_\tau = \frac{\int_0^\infty \tau\, \phi_g(\tau)\, d\tau}{\int_0^\infty \phi_g(\tau)\, d\tau}$$
- RMS delay spread: $$\sigma_\tau = \sqrt{\frac{\int_0^\infty (\tau - \mu_\tau)^2 \phi_g(\tau)\, d\tau}{\int_0^\infty \phi_g(\tau)\, d\tau}}$$

Multipath Intensity Profile (Cont.)

- Middle profile: W_x
  - Contains x% of the total power in the profile

$$W_x = \tau_3 - \tau_1$$

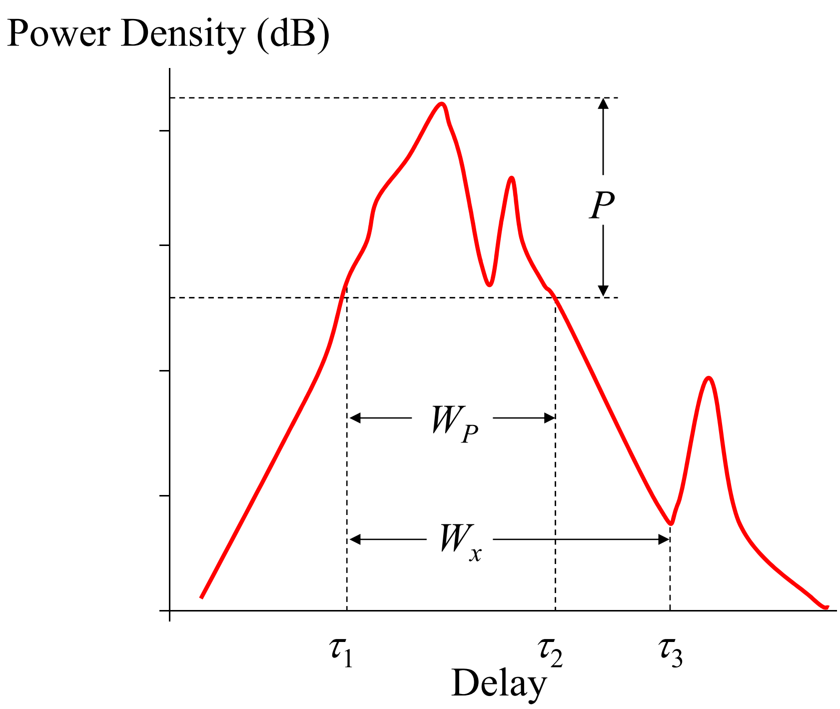
$$\int_0^{\tau_1} \phi_g(\tau)\, d\tau = \int_{\tau_3}^{\infty} \phi_g(\tau)\, d\tau$$

$$\int_{\tau_1}^{\tau_3} \phi_g(\tau)\, d\tau = x\% \int_0^{\infty} \phi_g(\tau)\, d\tau$$

- Difference in delay: W_P
  - The delay profile rises to a value P dB below the maximum value: τ_1
  - The delay profile drops to a value P dB below the maximum value: τ_2

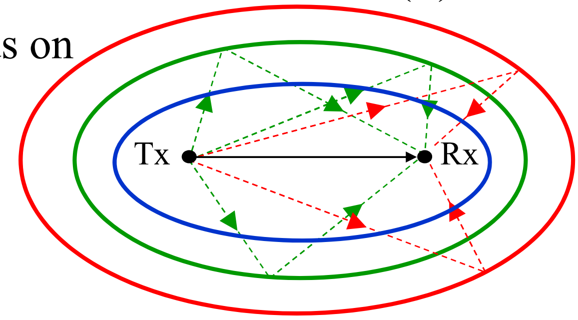
$$W_P = \tau_2 - \tau_1$$

Multipath Intensity Profile (Cont.)



Multipath Intensity Profile (Cont.)

- The power delay profiles play a key role in determining the need of an adaptive equalizer
- If the delay spread exceeds **10% to 20%** of the symbol duration
  - An adaptive equalizer is required
- Delay spread diminish (↓) with the decrease in cell size (↓)
- The delay spread strongly depends on the environment (and frequency):
  - Urban, suburban, open area
  - Macrocellular: 1 ~ 10 μs
  - In building: 30 ~ 60 *n*s



- The value of delay spread impacts on the transmission rate
  - Under the considerations of **complexity** and **performance**

Coherence Bandwidth

- For WSSUS, the autocorrelation function of $T(t, f)$ is $\phi_T(\Delta t;\Delta f)$: spaced-frequency spaced-time correlation function
- For $\Delta t = 0$, $\phi_T(0;\Delta f) = \phi_T(\Delta f)$ measures the frequency correlation of the channel (depending on the multipath intensity profile)
- **Coherence Bandwidth** B_c:
  - The smallest value of Δf for which $\phi_T(\Delta f)$ equals some suitable correlation coefficient, such as 0.5
- $\phi_g(\tau)$ and $\phi_T(\Delta f)$ are Fourier transform pair

$$B_c \propto \frac{1}{\sigma_\tau}$$

  - σ_τ: the rms delay spread

Coherence Bandwidth (Cont.)

- For frequency non-selective fading:
  - The transmission bandwidth ($1/T_s$) is smaller than B_c
  - The symbol duration $T_s >> \sigma_\tau$
- For frequency selective fading:
  - The transmission bandwidth ($1/T_s$) is larger than or equivalent to B_c
  - The symbol duration $T_s \cong \sigma_\tau$ or $T_s < \sigma_\tau$

Doppler Spread and Coherence Time

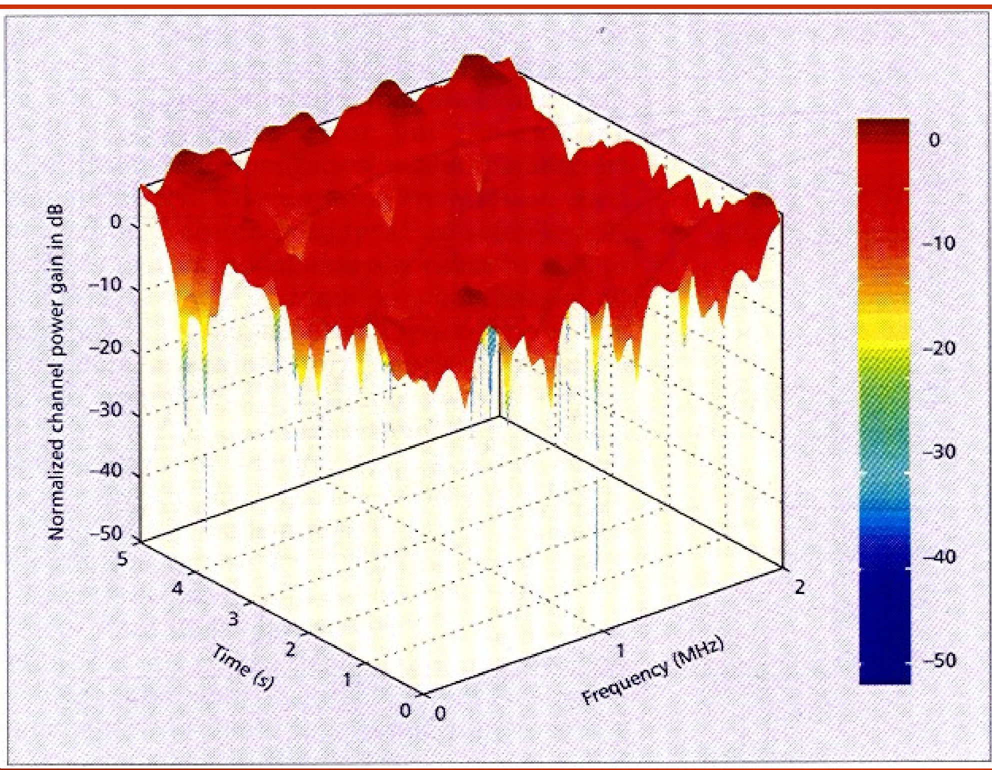
- For WSSUS, the autocorrelation function of $H(\nu, f)$: $\phi_H(\nu\,; \Delta f)$
- Doppler power spectral density: For $\Delta f = 0$, $\phi_H(\nu\,; 0) = \phi_H(\nu)$ shows the power density
  - The average power at the channel output as a function of Doppler frequency ν
- **Doppler Spread** B_d:
  - The range of values over which $\phi_H(\nu)$ is significant
- $\phi_H(\nu)$ and $\phi_T(\Delta t)$ are Fourier transform pair
  - The inverse of the Doppler spread B_d gives a measure of the coherence time T_c

$$T_c \approx \frac{1}{B_d}$$

Doppler Spread and Coherence Time (Cont.)

- **Coherence time** (corresponding to the average fade duration):
  - Can be used to evaluate the performance of coding and interleaving techniques
  - Coding and interleaving $\Rightarrow$ **time diversity**
- The duration of interleaving should much larger than the coherence time
- The Doppler spread and the coherence time depend directly on the **velocity** of a moving MS

Fading Channel



Question

- **Question:**
  - For what characteristics will a channel have a flat frequency response?
  - For what characteristics will a channel have a large time domain fading correlation?
  - What decides the frequency-domain/time-domain fading characteristics?
- Small delay spread $\Rightarrow$ Large coherence bandwidth B_c
  - A simple propagation environment
- Small Doppler spread $B_d \Rightarrow$ Large coherence time T_c
  - Low user mobility
- Frequency-domain: **propagation environments**
- Time-domain: **user mobility**

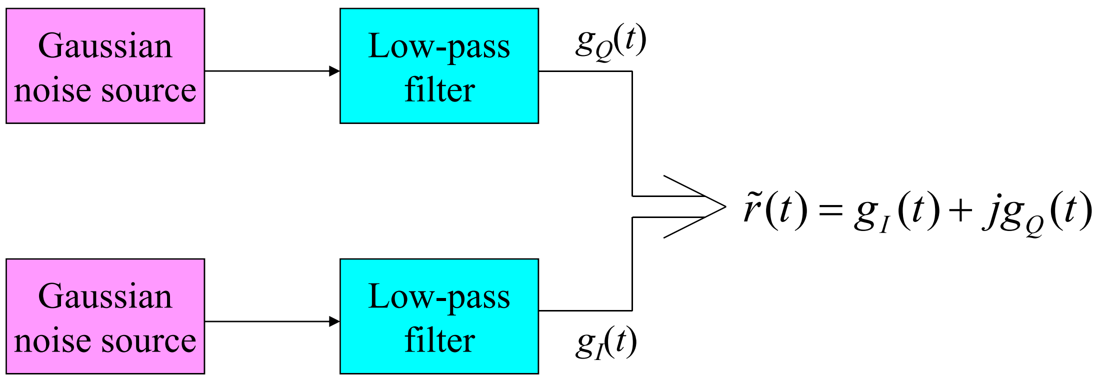
Laboratory Simulation

Simulation of Multipath-Fading Channels

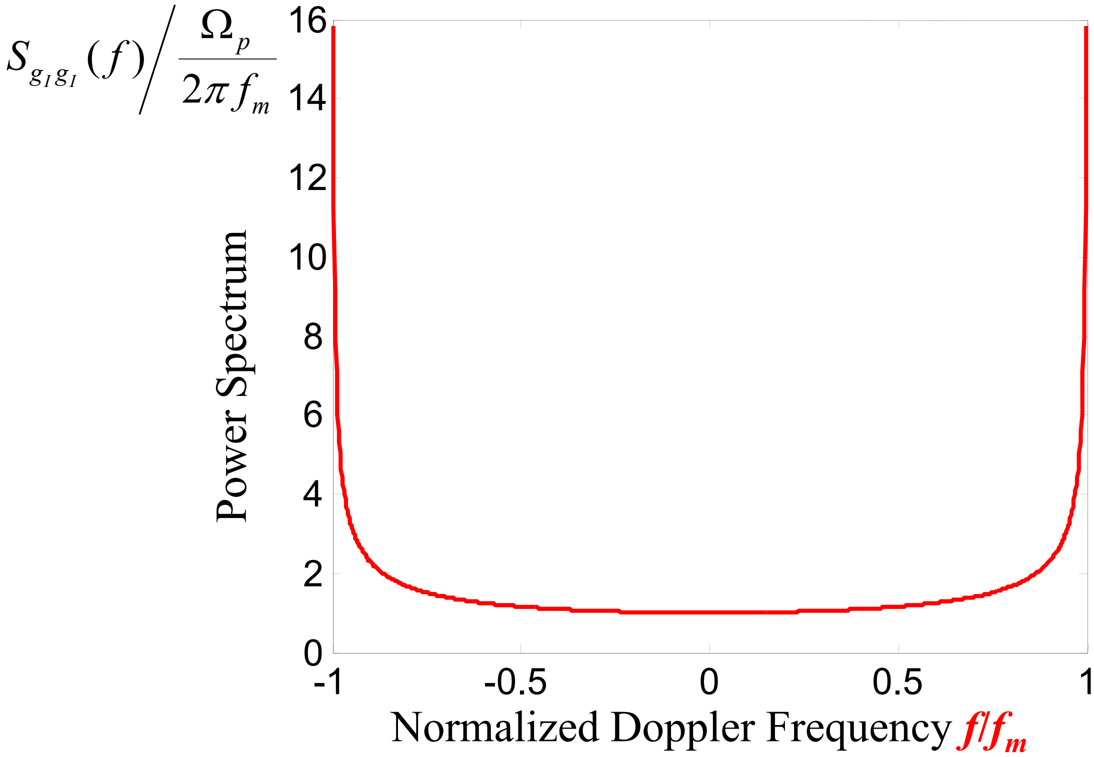
- The multipath fading channel simulator:
  - Filtered Gaussian Noise Method
  - Jakes' Method
  - Wide-band multipath-fading channels

Filtered Gaussian Noise Method

- Gaussian noise sources:
  - Zero mean: Rayleigh fade envelope
  - Non-zero mean: Ricean fade envelope
  - The two different noise sources must have the same PSD
- Low-pass filter: the output PSD should have the actual Doppler PSD of the multipath fading channel



Doppler Spectrum of Multipath Fading Channel



Filtered Gaussian Noise Method (Cont.)

- In order to approximate the Doppler spectrum of the multipath fading channel, a **high order filter** is required
  - $\Rightarrow$ Long impulse response
  - $\Rightarrow$ Significantly increase the run times
- Advantage: different paths are uncorrelated (if the Gaussian noise sources are uncorrelated)
- Disadvantage: hard to provide correct autocorrelation (a high order filter is required)
- If the noise sources have **constant** power spectral densities of $\Omega_p/2$ and the low-pass filters have transfer function $H(f)$
  - We have $S_{g_I g_I}(f) = S_{g_Q g_Q}(f) = \frac{\Omega_p}{2}|H(f)|^2$

$$S_{g_I g_Q}(f) = S_{g_Q g_I}(f) = 0$$

Filtered Gaussian Noise Method (Cont.)

- Let $g_{I,k} \equiv g_I(kT)$ and $g_{Q,k} \equiv g_Q(kT)$ represent the real and imaginary parts of the complex envelope at epoch k, where T is the simulation step size
- Using a **first-order** low-pass digital filter

$$(g_{I,k+1}, g_{Q,k+1}) = \zeta(g_{I,k}, g_{Q,k}) + (1-\zeta)(w_{1,k}, w_{2,k})$$

  - where $w_{1,k}$ and $w_{2,k}$ are **independent** zero-mean Gaussian random variables

$$E\left[g_{I,k}g_{I,k}\right] = \zeta^2 E\left[g_{I,k-1}g_{I,k-1}\right] + (1-\zeta)^2\sigma^2 \Rightarrow \sigma_{g_I}^2 = \sigma_{g_Q}^2 = \frac{1-\zeta}{1+\zeta}\sigma^2$$

  - σ^2: the variance of $w_{1,k}$ and $w_{2,k}$, and

Auto-correlation

$$\phi'_{g_I g_I}(n) = \phi'_{g_Q g_Q}(n) = E[g_{I,k}g_{I,k+n}] = \frac{1-\zeta}{1+\zeta}\sigma^2\zeta^{|n|}$$

Cross-correlation $\longrightarrow$ $\phi'_{g_I g_Q}(n) = \phi'_{g_Q g_I}(n) = 0$

Filtered Gaussian Noise Method (Cont.)

- The values of σ^2 and ζ should be specified
- For isotropic scattering, the ideal auto-correlation is

$$\phi_{g_I g_I}(n) = \frac{\Omega_p}{2} J_0(2\pi f_m nT)$$

$$\boxed{u[n] = \begin{cases} +1, & n \geq 0 \\ 0, & n < 0 \end{cases}}$$

- Taking DFT on $\phi'_{g_I g_I}(n)$

Different

$$\phi'_{g_I g_I}(n) = \frac{1-\zeta}{1+\zeta}\sigma^2\zeta^{|n|} = \frac{1-\zeta}{1+\zeta}\sigma^2\left(\zeta^n u[n] + \zeta^{-n}u[-n] - \delta[n]\right)$$

$$\zeta^n u[n] \overset{\mathbb{F}}{\longleftrightarrow} \frac{1}{1-\zeta e^{-j2\pi f t}},\ \zeta^{-n}u[-n] \overset{\mathbb{F}}{\longleftrightarrow} \frac{1}{1-\zeta e^{j2\pi f t}},\ \delta[n] \overset{\mathbb{F}}{\longleftrightarrow} 1$$

$$S'_{g_I g_I}(f) = \mathbb{F}\left\{\phi'_{g_I g_I}(n)\right\} = \frac{(1-\zeta)^2\sigma^2}{1+\zeta^2 - 2\zeta\cos 2\pi fT}$$

Filtered Gaussian Noise Method (Cont.)

$$S'_{g_I g_I}(f) = \frac{(1-\zeta)^2 \sigma^2}{1+\zeta^2 - 2\zeta\cos 2\pi f T}$$

- Set the 3 dB point of $S'_{g_I g_I}(f)$ to $f_m/4$, $\boxed{S'_{g_I g_I}(f_m/4) = S'_{g_I g_I}(0)/2}$ we have

$$\zeta^2 - 2\zeta\left(2-\cos\left(\pi f_m T/2\right)\right)+1=0$$

$$\zeta = 2-\cos(\pi f_m T/2) - \sqrt{(2-\cos(\pi f_m T/2))^2 - 1}$$

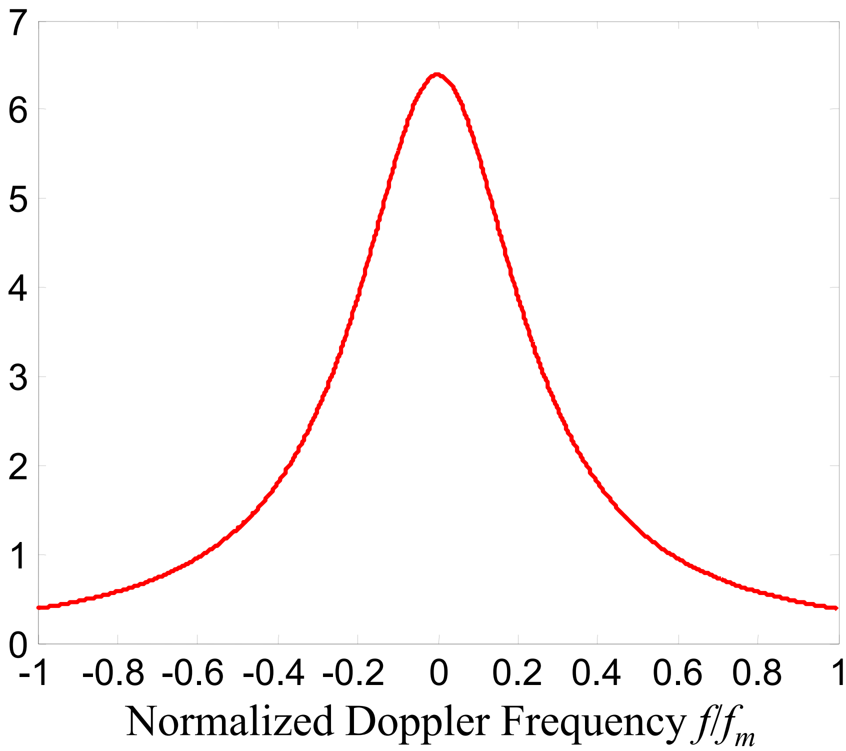
- To normalized the mean square envelope to Ω_p

$$\sigma_{g_I}^2 = \frac{1-\zeta}{1+\zeta}\sigma^2 = \frac{\Omega_p}{2} \Rightarrow \sigma^2 = \frac{1+\zeta}{(1-\zeta)}\frac{\Omega_p}{2}$$

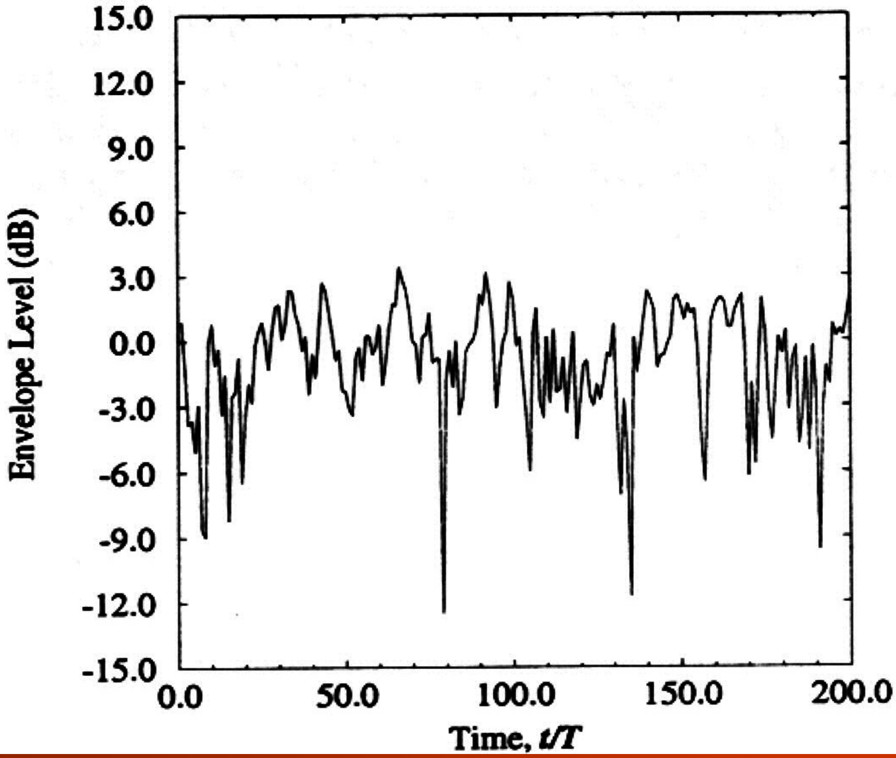
Filtered Gaussian Noise Method (Cont.)

$S'_{g_I g_I}(f)$

Obtained by Filtered Gaussian Noise method



Filtered Gaussian Noise Method (Cont.)



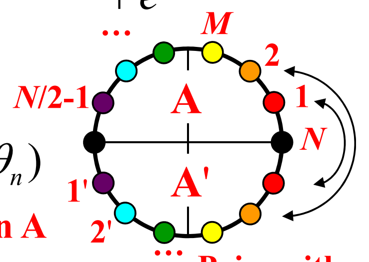
Sum of Sinusoids Method

- From
$$g(t) = \sum_{n=1}^{N} \alpha_n(t)\, e^{-j\phi_n(t)}$$
$$\phi_n(t) = 2\pi\left\{\left(f_c + f_{D,n}(t)\right)\tau_n(t) - f_{D,n}(t)t\right\}$$
- Assume that
  - The channel is stationary ($f_{D,n}(t) = f_{D,n}$, $\tau_n(t) = \tau_n$, $\alpha_n(t) = \alpha_n$)
  - Equal strength of multipath components ($\alpha_n = 1, \forall n$)
$$g(t) = \sum_{n=1}^{N} e^{j2\pi\left[f_m t\cos\theta_n - (f_c + f_m\cos\theta_n)\tau_n\right]} = \sum_{n=1}^{N} e^{j(2\pi f_m t\cos\theta_n + \hat{\phi}_n)}$$
- For an isotropic scattering environment, we can assume that the **incident angles** are uniformly distributed
$$\theta_n = \frac{2\pi n}{N}, \quad n = 1, 2, \cdots, N$$

Sum of Sinusoids Method (Cont.)

- Choose $N/2$ to be an odd integer, we can rewrite $g(t)$ as

$$g(t)=\sum_{n=1}^{N/2-1}\Big[\underbrace{e^{j(2\pi f_m t\cos\theta_n+\hat{\phi}_n)}}_{\mathbf{A}}+\underbrace{e^{-j(2\pi f_m t\cos\theta_n+\hat{\phi}_{-n})}}_{\mathbf{A'}}\Big]+e^{j(2\pi f_m t+\hat{\phi}_N)}+e^{-j(2\pi f_m t+\hat{\phi}_{-N})}$$



Pairs with the same freq.

- Some terms have the same freq. components

$$\underbrace{2\pi f_m t\cos(\theta_n+\pi)}_{\text{Term } N/2+n}=\underbrace{-2\pi f_m t\cos\theta_n}_{\text{Term } n \text{ in A'}}\xleftrightarrow{\text{same freq.}}\underbrace{=2\pi f_m t\cos(\pi-\theta_n)}_{\text{Term } N/2-n \text{ in A}}$$

- Combining the terms with the same freq., we have

$$g(t)=\underbrace{\sqrt{2}}_{\text{Maintain the same total power}}\sum_{n=1}^{M}\Big[e^{j(2\pi f_m t\cos\theta_n+\hat{\phi}_n)}+e^{-j(2\pi f_m t\cos\theta_n+\hat{\phi}_{-n})}\Big]+e^{j(2\pi f_m t+\hat{\phi}_N)}+e^{-j(2\pi f_m t+\hat{\phi}_{-N})}$$

$$M=\frac{1}{2}\left(\frac{N}{2}-1\right)$$

- For the same freq. components, the phases are now set to be the same $\Rightarrow$ Correlation is introduced into the phases

Sum of Sinusoids Method (Cont.)

$$g(t)=\sqrt{2}\sum_{n=1}^{M}\Big[e^{j(2\pi f_m t\cos\theta_n+\hat{\phi}_n)}+e^{-j(2\pi f_m t\cos\theta_n+\hat{\phi}_{-n})}\Big]+e^{j(2\pi f_m t+\hat{\phi}_N)}+e^{-j(2\pi f_m t+\hat{\phi}_{-N})}$$

- If we further adopt the constraint that $\hat{\phi}_n=-\hat{\phi}_{-n}$, we have

$$\boxed{\begin{aligned}\cos(x+y)&=\cos(x)\cos(y)-\sin(x)\sin(y)\\ \sin(x+y)&=\sin(x)\cos(y)+\cos(x)\sin(y)\end{aligned}}$$

$$\boxed{2\pi f_n t=2\pi f_m t\cos\theta_n}$$

$$\begin{aligned}g(t)&=g_I(t)+jg_Q(t)\\ &=\sqrt{2}\Big\{\Big[2\sum_{n=1}^{M}\cos\beta_n\cos\underline{2\pi f_n t}+\sqrt{2}\cos\alpha\cos 2\pi f_m t\Big]\\ &\qquad+j\Big[2\sum_{n=1}^{M}\sin\beta_n\cos 2\pi f_n t+\sqrt{2}\sin\alpha\cos 2\pi f_m t\Big]\Big\}\end{aligned}$$

  - where $\alpha=\hat{\phi}_N=-\hat{\phi}_{-N}$, $\beta_n=\hat{\phi}_n=-\hat{\phi}_{-n}$
- Only **(*M*+1)** independent frequency oscillators are required
  - There are $(M+1)$ different frequencies

Sum of Sinusoids Method (Cont.)

- Considering the channel statistics

$$E\left[g_I^2(t)\right] = 2\sum_{n=1}^{M}\cos^2\beta_n + \cos^2\alpha = M + \cos^2\alpha + \sum_{n=1}^{M}\cos 2\beta_n$$

$$E\left[g_Q^2(t)\right] = 2\sum_{n=1}^{M}\sin^2\beta_n + \sin^2\alpha = M + \sin^2\alpha - \sum_{n=1}^{M}\cos 2\beta_n$$

$$E\left[g_I(t)g_Q(t)\right] = 2\sum_{n=1}^{M}\sin\beta_n\cos\beta_n + \sin\alpha\cos\alpha$$

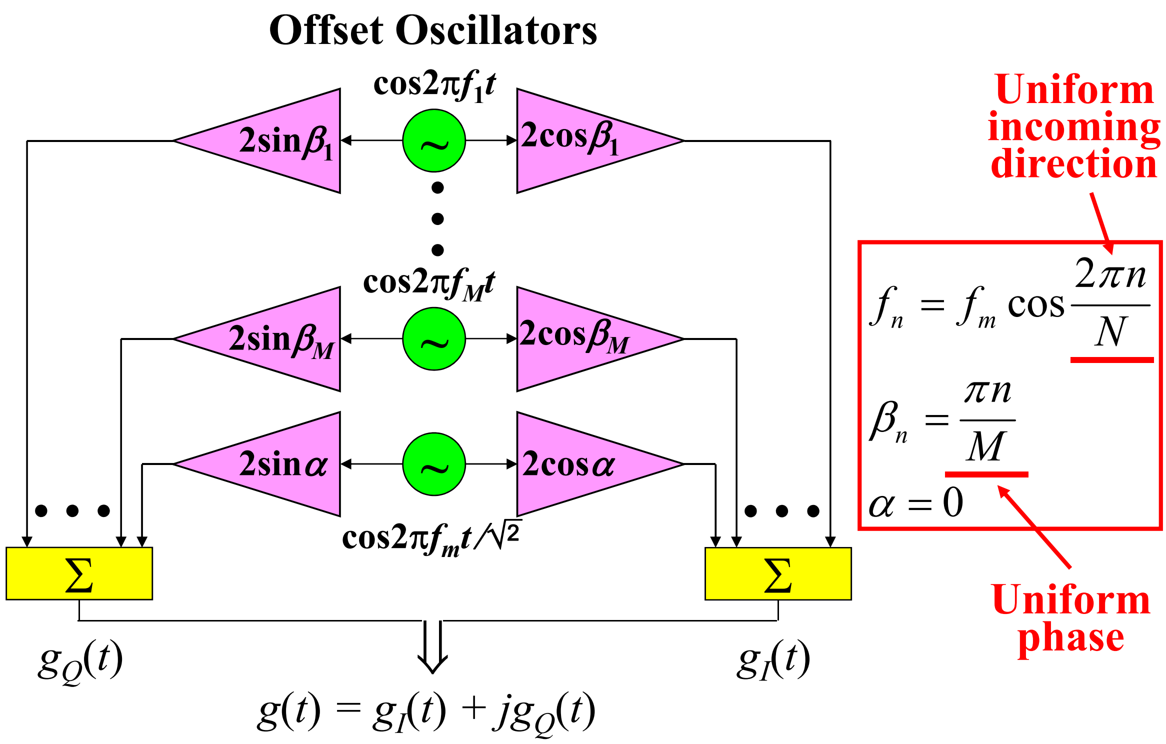
- It is desirable that

$$E\left[g_I^2(t)\right] = E\left[g_Q^2(t)\right], \quad E\left[g_I(t)g_Q(t)\right] = 0$$

- Choose the parameters $\beta_n = \dfrac{\pi n}{M}, \quad \alpha = 0$

$$E\left[g_I^2(t)\right] = M + 1, \quad E\left[g_Q^2(t)\right] = M, \quad E\left[g_I(t)g_Q(t)\right] = 0$$

Sum of Sinusoids Method (Cont.)



$$f_n = f_m \cos\frac{2\pi n}{N}$$

$$\beta_n = \frac{\pi n}{M}$$

$$\alpha = 0$$

Sum of Sinusoids Method (Cont.)

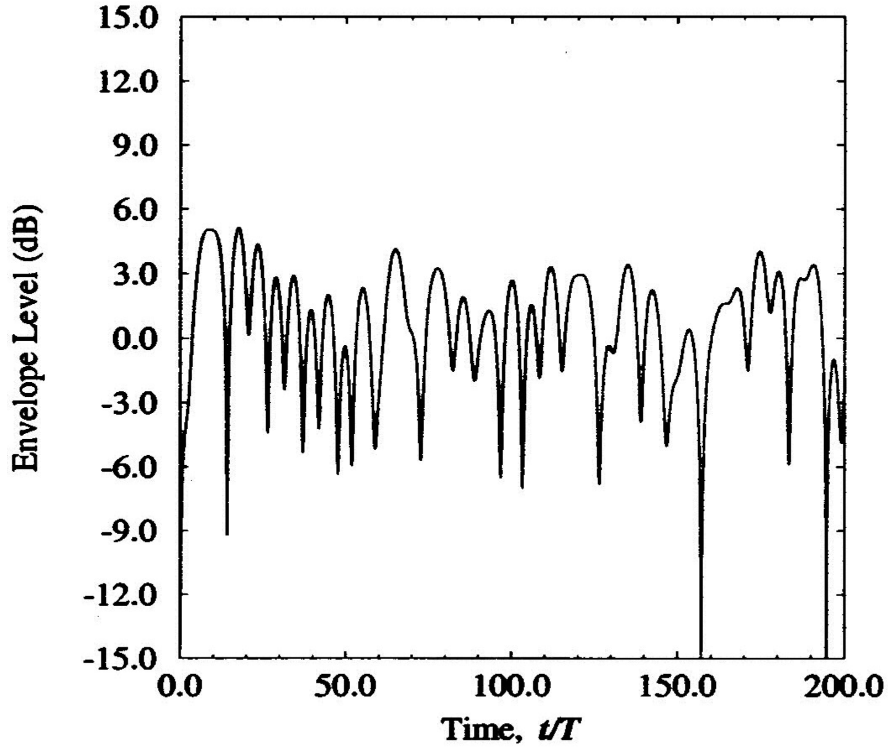
- If the last term of $g_I(t)$ is ignored, we have

$$E\left[g_I^2(t)\right]=M, \quad E\left[g_Q^2(t)\right]=M, \quad E\left[g_I(t)g_Q(t)\right]=0$$

  – when $\beta_n = \dfrac{\pi n}{M}, \quad \alpha = 0$
- Advantage: the **autocorrelation** of inphase and quadrature components reflect an **isotropic scattering** environment with a reasonable complexity
- The channel model output is a **deterministic process**
  – No random number generator is applied

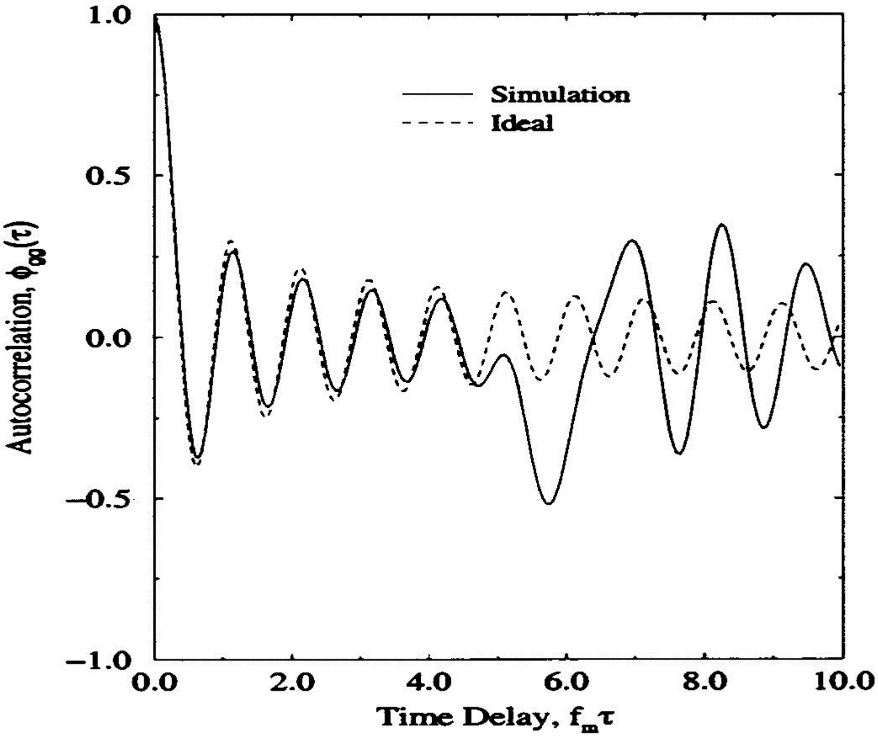
Sum of Sinusoids Method (Cont.)

- $M = 8$



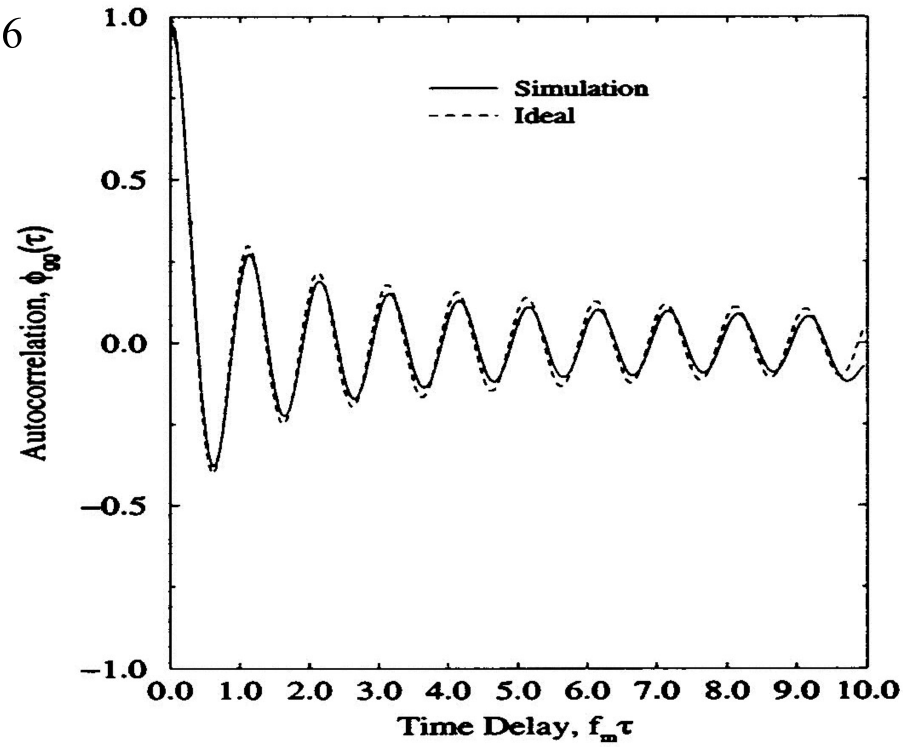
Sum of Sinusoids Method (Cont.)

- $M = 8$



Sum of Sinusoids Method (Cont.)

- $M = 16$



Wide-Band Multipath-Fading Channels

- For wide-band communication systems, the time-domain resolution is increased and multiple paths can be resolved
- τ–spaced model:
  - Model the channel by a tapped delay line
  - Assume a number of discrete paths at different delays

$$\tilde{r}(t) = \sum_{i=1}^{\ell} g_i(t)\tilde{s}(t-\tau_i)$$

  - $g_i(t)$ and τ_i are the tap gain and delay of the i-th path

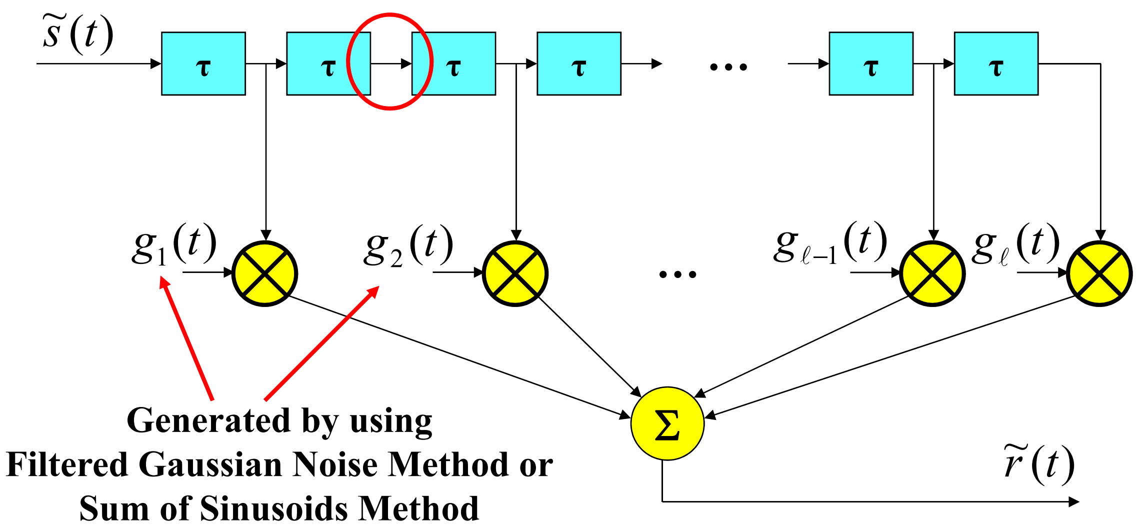
$$g(t,\tau) = \sum_{i=1}^{\ell} g_i(t)\delta(t-\tau_i)$$

  - The tap gain and tap delay vectors

$$\mathbf{g}(t) = \left(g_1(t),\, g_2(t),\, \cdots,\, g_\ell(t)\right)$$
$$\boldsymbol{\tau} = \left(\tau_1,\, \tau_2,\, \cdots,\, \tau_\ell\right)$$

Wide-Band Multipath-Fading Channels (Cont.)

- The path delays are multiples of some small number τ



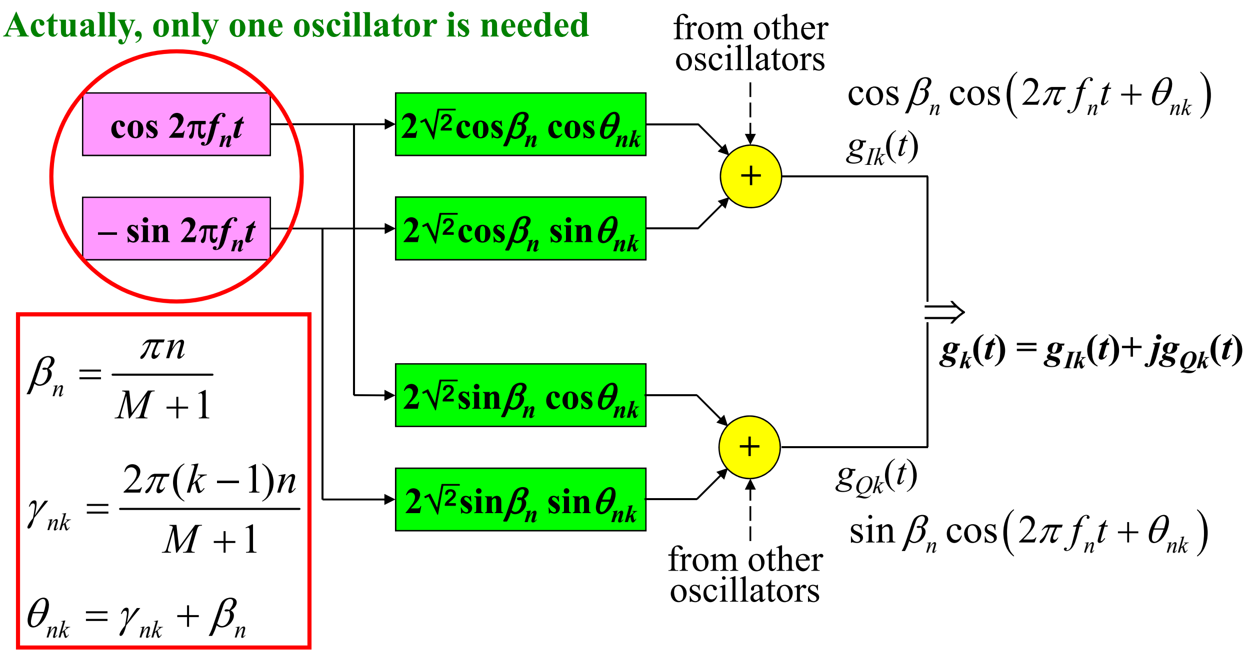
Multiple Faded Envelopes

- In many cases, it is desirable to generate multiple envelopes with **uncorrelated fading** (i.e., different paths with resolvable delays)
  - Generate up to $\boldsymbol{M}$ fading envelopes by using the same $\boldsymbol{M}$ frequency oscillators
- Give the n-th oscillator, $1 \le n \le M$, an additional phase shift $\theta_{nk} = \gamma_{nk} + \beta_n$, $1 \le k \le M$, where k is the index of fading envelopes
- An additional constraint: the multiple faded envelopes should be **uncorrelated**
  - Choose appropriate values of γ_{nk} and β_n
- The k-th fading envelope is (ignore the last term of $g_I(t)$)

$$g_k(t) = 2\sqrt{2}\sum_{n=1}^{M}\left(\cos\beta_n + j\sin\beta_n\right)\underline{\cos\left(2\pi f_n t + \theta_{nk}\right)}$$

Multiple Faded Envelopes (Cont.)

- By using two quadrature frequency oscillators



Multiple Faded Envelopes (Cont.)

- Choose the parameters with the objective yielding uncorrelated waveforms

$$\beta_n = \frac{\pi n}{M+1}, \quad \gamma_{nk} = \frac{2\pi(k-1)n}{M+1}, \quad n = 1, 2, \cdots, M$$

- Significant cross-correlation between the different generated fading envelopes (**without modification**)
- A modification that uses orthogonal **Walsh-Hadamard** codewords to decorrelate the fading envelopes is applied
  - $A_k(n)$: the k-th row of Hadamard matrix $\mathbf{H}_M$
  - **$A_k(n)$: +1 ("0") or –1 ("1")**

$$g_k(t) = 2\sqrt{2}\sum_{n=1}^{M} A_k(n)\left(\cos\beta_n + j\sin\beta_n\right)\cos\left(2\pi f_n t + \theta_{nk}\right)$$

Multiple Faded Envelopes (Cont.)

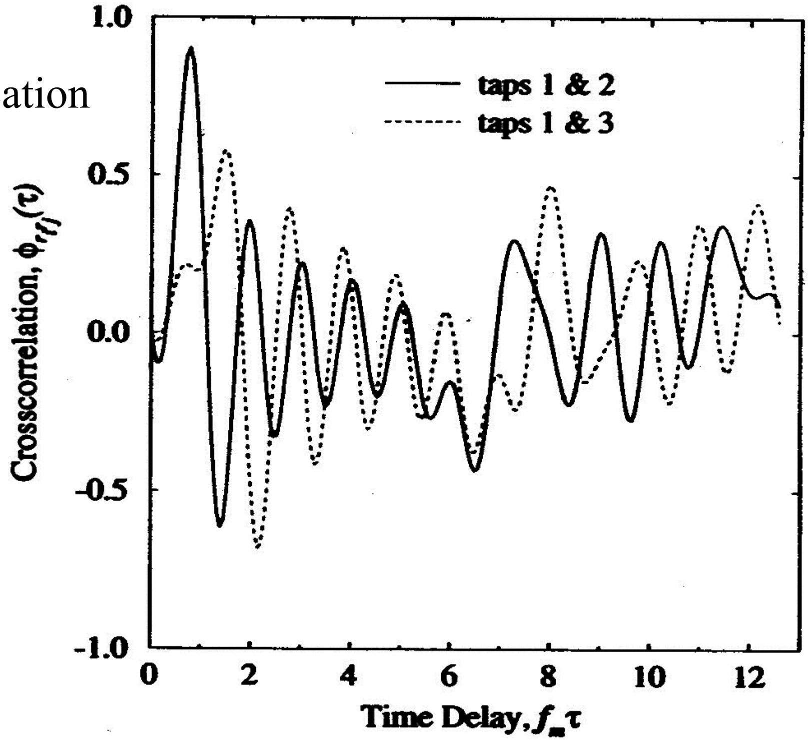
- Walsh-Hadamard codes:
  - It is an **orthogonal code set**
  - The cross-correlation between different codes is zero
- The code period of Walsh codes must be a power of 2
  - The code length must be 2, 4, 8, 16, …

$$\mathbf{H}_1 = [0] \quad \mathbf{H}_{2^n} = \begin{bmatrix} \mathbf{H}_{2^{(n-1)}} & \mathbf{H}_{2^{(n-1)}} \\ \mathbf{H}_{2^{(n-1)}} & \overline{\mathbf{H}_{2^{(n-1)}}} \end{bmatrix}$$

$$\mathbf{H}_2 = \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix}; \quad \mathbf{H}_4 = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 1 \\ 0 & 1 & 1 & 0 \end{bmatrix}; \quad \mathbf{H}_8 = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 & 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 1 & 0 & 0 & 1 & 1 \\ 0 & 1 & 1 & 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 & 1 & 1 & 1 \\ 0 & 1 & 0 & 1 & 1 & 0 & 1 & 0 \\ 0 & 0 & 1 & 1 & 1 & 1 & 0 & 0 \\ 0 & 1 & 1 & 0 & 1 & 0 & 0 & 1 \end{bmatrix};$$

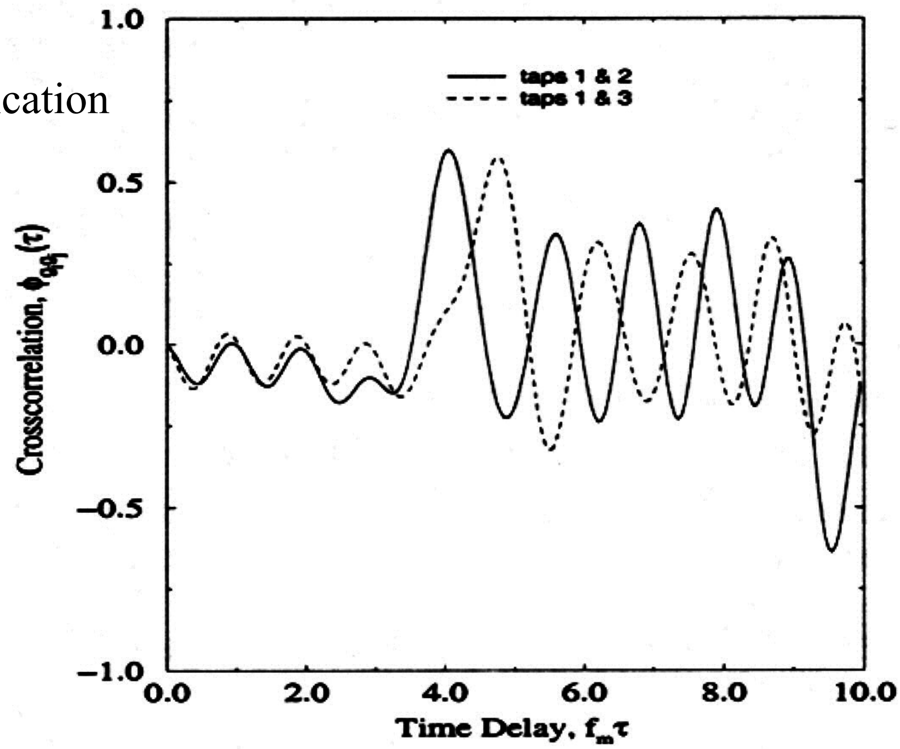
Multiple Faded Envelopes (Cont.)

- $M = 8$
- Without modification



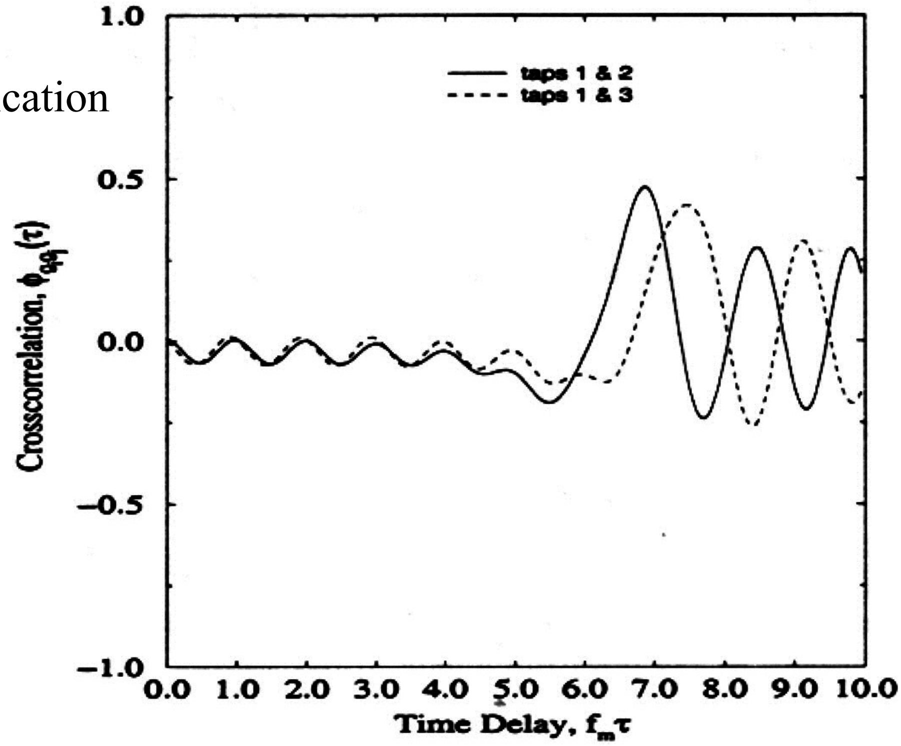
Multiple Faded Envelopes – Modified

- $M = 8$
- With modification



Multiple Faded Envelopes – Modified (Cont.)

- $M = 16$
- With modification



Shadowing

Shadowing

- Ω_v: the mean envelope level, $\Omega_v=E[\alpha(t)]$
  - where $\alpha(t)$ is Rayleigh or Ricean distributed
  - The local mean: averaged over a few wavelengths
- Ω_p: the mean squared envelope level, $\Omega_p=E[\alpha^2(t)]$
- Ω_v and Ω_p are random variables due to shadow variations that caused by
  - **Macrocell:** large terrain features (buildings, hills)
  - **Microcell:** small objects (vehicles, human)
- Ω_v and Ω_p follow the log-normal distributions

Linear Scale

$$p_{\Omega_v}(x)=\frac{2}{x\sigma_\Omega\xi\sqrt{2\pi}}\exp\left[-\frac{(10\log_{10}x^2-\mu_{\Omega_v(\mathrm{dBm})})^2}{2\sigma_\Omega^2}\right]$$

$$p_{\Omega_p}(x)=\frac{1}{x\sigma_\Omega\xi\sqrt{2\pi}}\exp\left[-\frac{(10\log_{10}x-\mu_{\Omega_p(\mathrm{dBm})})^2}{2\sigma_\Omega^2}\right]$$

Shadowing (Cont.)

- where $\xi=\ln 10/10$ and

$$\mu_{\Omega_v(\mathrm{dBm})}=30+10E[\log_{10}\Omega_v^2];\quad \mu_{\Omega_p(\mathrm{dBm})}=30+10E[\log_{10}\Omega_p]$$

- $\Omega_{v(\mathrm{dBm})}$ and $\Omega_{p(\mathrm{dBm})}$ have the Gaussian densities
  - The mean is determined by the propagation **path loss**

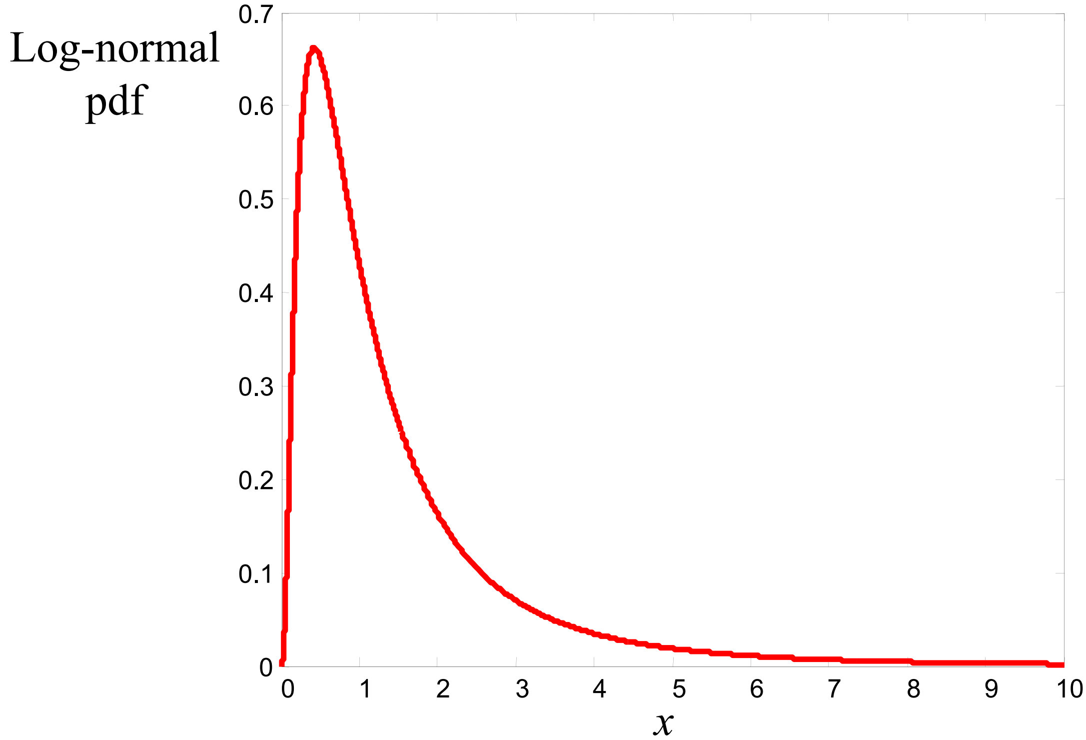
dB Scale

$$p_{\Omega_v(\mathrm{dBm})}(x)=\frac{1}{\sqrt{2\pi}\sigma_\Omega}\exp\left[-\frac{(x-\mu_{\Omega_v(\mathrm{dBm})})^2}{2\sigma_\Omega^2}\right]$$

$$p_{\Omega_p(\mathrm{dBm})}(x)=\frac{1}{\sqrt{2\pi}\sigma_\Omega}\exp\left[-\frac{(x-\mu_{\Omega_p(\mathrm{dBm})})^2}{2\sigma_\Omega^2}\right]$$

- The standard deviation of log-normal shadowing ranges:
  - Macrocell: **5 ~ 12 dB** with typical value σ_Ω = 8 dB
    - σ_Ω increases slightly with frequency ($\sigma_{1.8\mathrm{GHz}}=\sigma_{900\mathrm{MHz}}$+0.8dB)
  - Microcell: **4 ~ 13 dB**

Shadowing (Cont.)



Simulation of Shadowing

- A shadow simulator should account the spatial correlation
- One simple model: the log-normal shadowing is modeled as
  - A Gaussian white noise process
  - Filtered with a first-order low-pass filter

$$\Omega_{k+1(\mathrm{dBm})} = \zeta\, \Omega_{k(\mathrm{dBm})} + (1-\zeta) v_k$$

  - k: the location index
  - ζ: control the spatial correlation of the shadowing
  - v_k: a zero-mean Gaussian random variable, $\phi_{vv}(n) = \tilde{\sigma}^2 \delta(n)$
- The spatial autocorrelation function:

$$\phi_{\Omega_{(\mathrm{dBm})}\Omega_{(\mathrm{dBm})}}(n) = \frac{1-\zeta}{1+\zeta} \tilde{\sigma}^2 \zeta^{|n|}$$

$$\sigma_\Omega^2 = \phi_{\Omega_{(\mathrm{dBm})}\Omega_{(\mathrm{dBm})}}(0) = \frac{1-\zeta}{1+\zeta} \tilde{\sigma}^2$$

Simulation of Shadowing (Cont.)

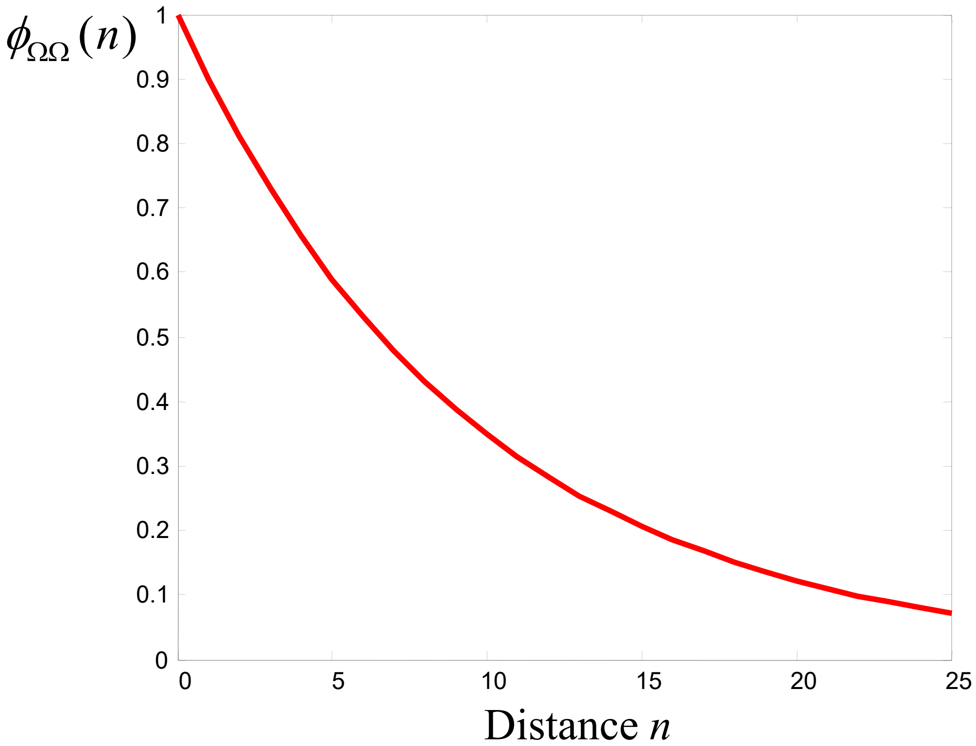
$$\phi_{\Omega_{(\mathrm{dBm})}\Omega_{(\mathrm{dBm})}}(n) = \sigma_\Omega^2 \zeta^{|n|}$$

- This approach generates shadows that decorrelated **exponentially with distance**
- If an MS is traveling with velocity v, the envelope is sampled for every T seconds, and ζ_D is the shadow correlation of spatial distance D m
  - Time difference $kT \Rightarrow$ spatial distance vkT

$$\phi_{\Omega_{(\mathrm{dBm})}\Omega_{(\mathrm{dBm})}}(k) \equiv \phi_{\Omega_{(\mathrm{dBm})}\Omega_{(\mathrm{dBm})}}(kT) = \sigma_\Omega^2 \zeta_D^{(vT/D)|k|}$$

  - $\zeta = \zeta_D^{(vT/D)}$
- Suburban 900 MHz: $\sigma_\Omega \approx 7.5$ dB with corr. 0.82 (100m)
- Microcell 1700 MHz: $\sigma_\Omega \approx 4.3$ dB with corr. 0.3 (10m)

Simulation of Shadowing (Cont.)



Path Loss Models

Free Space Path Loss Model

- Free space: the received signal power

$$\mu_{\Omega_p} = \Omega_t G_T G_R \left(\frac{\lambda_c}{4\pi d} \right)^2$$

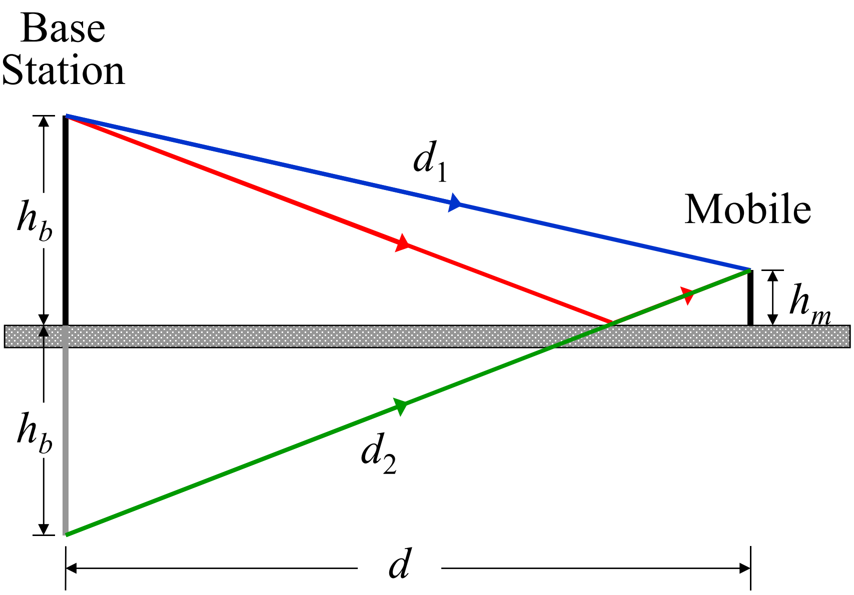
$$\mu_{\Omega_p\,(\mathrm{dB})} = 10\log_{10}\left(\Omega_t G_T G_R \left(\frac{\lambda_c}{4\pi d} \right)^2 \right)$$

$$\mu_{\Omega_p\,(\mathrm{dB})} = 10\log_{10}(\Omega_t G_T G_R / 16\pi^2) + 20\log_{10}\lambda_c - 20\log_{10} d$$

  - Ω_t: the transmission power
  - G_T and G_R: the transmitter and receiver antenna gains
  - λ_c: the wavelength
  - d: the radio path length

Mobile Radio Two-ray Path Loss Model

- Mobile radio environment (Two-ray model)



Mobile Radio Two-ray Path Loss Model (Cont.)

$$P_r = P_t(\tfrac{\lambda_c}{4\pi d})^2 \left|1+\alpha_v e^{j\Delta\phi}\right|^2$$

$$= P_t(\tfrac{\lambda_c}{4\pi d})^2 \left|1-\cos\Delta\phi - j\sin\Delta\phi\right|^2$$

$$= P_t(\tfrac{\lambda_c}{4\pi d})^2 \times 2(1-\cos\Delta\phi)$$

$$= P_t(\tfrac{\lambda_c}{4\pi d})^2 \times 4\sin^2\tfrac{\Delta\phi}{2}$$

α_v : reflection coefficient
$\alpha_v = -1$, for mobile environment
$\Delta\phi$: phase difference

$$\Delta\phi = \tfrac{2\pi}{\lambda_c}\Delta d, \text{ and } \Delta d = d_2 - d_1$$

$$d_1 = \sqrt{(h_b - h_m)^2 + d^2}, \text{ and } d_2 = \sqrt{(h_b + h_m)^2 + d^2}$$

$$d_2^2 - d_1^2 = 2d_1\Delta d + \Delta d^2 = 4h_b h_m$$

$$\Rightarrow \Delta d \approx 2h_b h_m / d, \quad \Delta\phi = \tfrac{4\pi h_b h_m}{\lambda_c d}$$

$$\left(\because \begin{cases} d_1 \approx d \\ \Delta d^2 \approx 0 \end{cases} \text{for } d >> 0 \right)$$

$$\Rightarrow P_r = P_t(\tfrac{\lambda_c}{4\pi d})^2 \times 4\sin^2(\tfrac{2\pi h_b h_m}{\lambda_c d})$$

Mobile Radio Two-ray Path Loss Model (Cont.)

- The received signal power is

$$\mu_{\Omega_p} = 4\Omega_t \left(\frac{\lambda_c}{4\pi d}\right)^2 G_T G_R \sin^2\left(\frac{2\pi h_b h_m}{\lambda_c d}\right)$$

- When $d >> h_b\, h_m$, $\sin x \approx x$

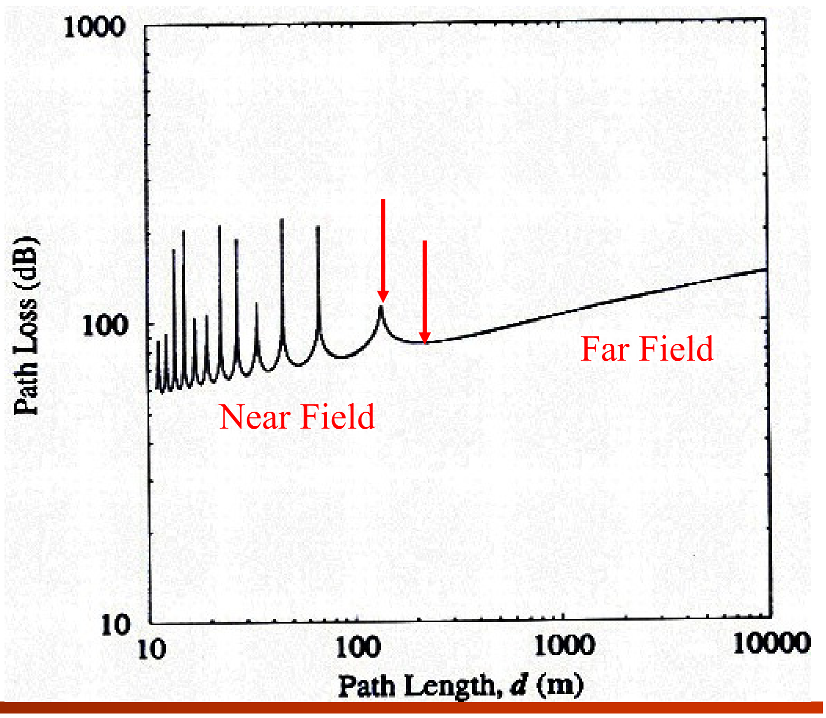
$$\mu_{\Omega_p} = \Omega_t G_T G_R \left(\frac{h_b h_m}{d^2}\right)^2$$

- The differences to the free space model are:
  - The path loss is not frequency dependent
  - The signal power decays with the 4th power of the distance
- The path loss is independent of Ω_t, G_T, and G_R

$$L_{p(\text{dB})} = 10\log_{10}\left\{\frac{\Omega_t G_T G_R}{\mu_{\Omega_p}}\right\} = -10\log_{10}\left\{4\left(\frac{\lambda_c}{4\pi d}\right)^2 \sin^2\left(\frac{2\pi h_b h_m}{\lambda_c d}\right)\right\} \text{dB}$$

Mobile Radio Two-ray Path Loss Model (Cont.)

- The last **local maximum** of the path loss occurs when $2\pi h_b h_m / \lambda_c d = \pi$
- The last **local minimum** of the path loss occurs when $2\pi h_b h_m / \lambda_c d = \pi/2$



Path Loss in Macrocells

- The path loss models used in macrocell applications are **empirical** models
  - The environment is **too complex** to obtained a completely theoretical-based model
  - Obtained by curve fitting based on the experimental data
- For 900 MHz cellular systems, the most common used path loss model is
  - **Okumura-Hata's model**
    - Empirical data was collected by Okumura (in Tokyo)
    - Modeled by Hata

Okumura-Hata's Model

- f_c: 150~1500MHz, d: >1Km, h_b: 30~200m, h_m: 1~10m

$$L_{p(\mathrm{dB})} = \begin{cases} A + B\log_{10}(d) & \text{for urban area} \\ A + B\log_{10}(d) - C & \text{for suburban area} \\ A + B\log_{10}(d) - D & \text{for open area} \end{cases}$$

  - where

$$A = 69.55 + 26.16\log_{10}(f_c) - 13.82\log_{10}(h_b) - a(h_m)$$
$$B = 44.9 - 6.55\log_{10}(h_b)$$
$$C = 5.4 + 2\left[\log_{10}(f_c/28)\right]^2$$
$$D = 40.94 + 4.78\left[\log_{10}(f_c)\right]^2 - 18.33\log_{10}(f_c)$$

$$a(h_m) = \begin{cases} \left[1.1\log_{10}(f_c) - 0.7\right]h_m - \left[1.56\log_{10}(f_c) - 0.8\right], & \text{for medium or small city} \\ \begin{cases} 8.28\left[\log_{10}(1.54h_m)\right]^2 - 1.1, & \text{for } f_c \le 200 \text{ MHz} \\ 3.2\left[\log_{10}(11.75h_m)\right]^2 - 4.97, & \text{for } f_c \ge 400 \text{ MHz} \end{cases}, & \text{for large city} \end{cases}$$

Okumura-Hata's Model (Cont.)

- Another empirical model published by the CCIR:

$$L_{p(\mathrm{dB})} = A + B\log_{10}(d) - E$$

  - where

$$A = 69.55 + 26.16\log_{10}(f_c) - 13.82\log_{10}(h_b) - a(h_m)$$

$$B = 44.9 - 6.55\log_{10}(h_b)$$

$$E = 30 - 25\log_{10}(\text{\% of area covered by buildings: } 1\sim100)$$

$$a(h_m) = \left[1.1\log_{10}(f_c) - 0.7\right]h_m - \left[1.56\log_{10}(f_c) - 0.8\right]$$

- The parameter E accounts for the degree of urbanization
  - $E = 0$ when the area is covered by approximately 16 % of buildings

Okumura-Hata's Model (Cont.)

- **Another expression**
- f_c: 150~1500MHz, d: >1Km, h_b: 30~200m, h_m: 1~10m
- $P_O = A + B\log(d)$

  $= [69.55 + 26.16\log(f_c) - 13.82\log(h_b)]$

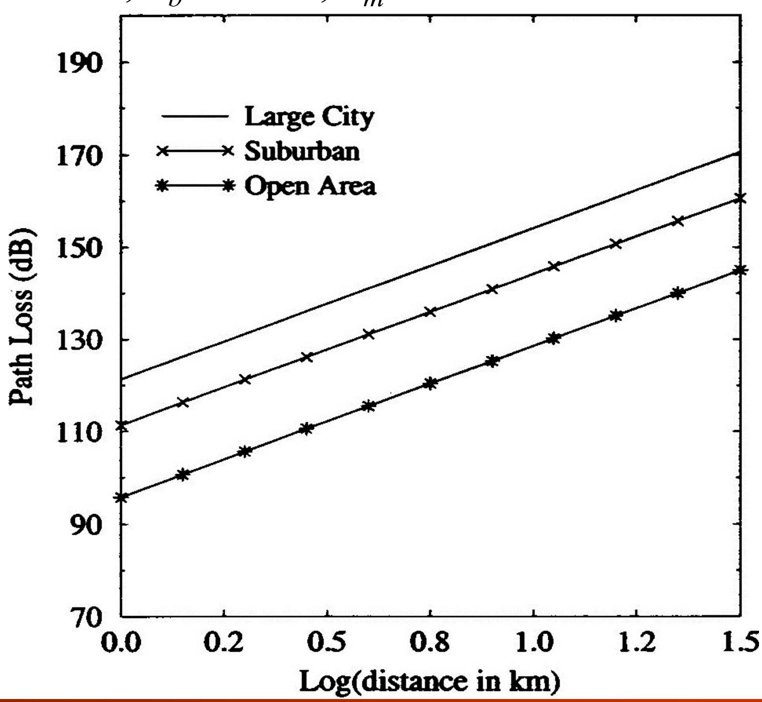
  $+ [44.9 - 6.55\log(h_b)]\log(d)$
- $a(h_m)$:
  - Large city:
    - $f_c < 200$MHz: $a(h_m) = 8.28[\log(1.54h_m)]^2 - 1.1$
    - $f_c > 400$MHz: $a(h_m) = 3.2[\log(11.75h_m)]^2 - 4.97$
  - Medium or Small city:
    - $a(h_m) = [1.1\log(f_c) - 0.7]h_m - [1.56\log(f_c) - 0.8]$

Okumura-Hata's Model (Cont.)

- **Distance correction factor**:
  - $d < 20$Km: cr(d) = 0
  - $d > 20$Km: cr(d) = (d – 20)[0.31081 + 0.1865log(h_b/100)]
  - $d > 64.36$Km: cr(d) = (d – 20)[0.31081 + 0.1865 log(h_b/100)] – 0.174(d – 64.36)
- Environment correction factor:
  - Urban area: ce(f_c) = 0
  - Suburban area: ce(f_c) = – 2[log(f_c/28)]^2 – 5.4
  - Open area: ce(f_c) = – 4.78[log(f_c)]^2 + 18.33 log(f_c) – 40.94
- **$P_L = P_O - a(h_m) + cr(d) + ce(f_c)$ dB**

Okumura-Hata's Model (Cont.)

- f_c = 900 MHz, h_b = 70 m, h_m = 1.5 m



Path Loss in Outdoor Macro-/Micro-cells

- For the PCS microcellular systems operating in 1800-2000 MHz frequency bands, the two common used path loss models are
  - COST231-Hata model (Macrocellular)
  - Two-slope model (Microcellular)

COST231-Hata Model

- Extend Okumura-Hata model for 1500-2000 MHz range
- f_c: 1500~2000 MHz, d: 1~20 Km, h_b: 30~200m, h_m: 1~10m

$$L_{p(\mathrm{dB})} = A + B\log_{10}(d) + C$$

Okumura-Hata's Model

$$A = 69.55 + 26.16\log_{10}(f_c) - 13.82\log_{10}(h_b) - a(h_m)$$
$$B = 44.9 - 6.55\log_{10}(h_b)$$

$$A = 46.3 + 33.9\log_{10}(f_c) - 13.82\log_{10}(h_b) - a(h_m)$$
$$B = 44.9 - 6.55\log_{10}(h_b)$$
$$C = \begin{cases} 0 & \text{medium city and suburban areas with moderate tree density} \\ 3 & \text{for metropolitan centers} \end{cases}$$

- Good accuracy for a path length larger than 1 km
- Should not be used for smaller ranges (near field)
  - The path loss becomes **highly dependent** upon the local topography

Two-Slope Model (Street Microcells)

- For a range less than 500m and the antenna height less than 20m

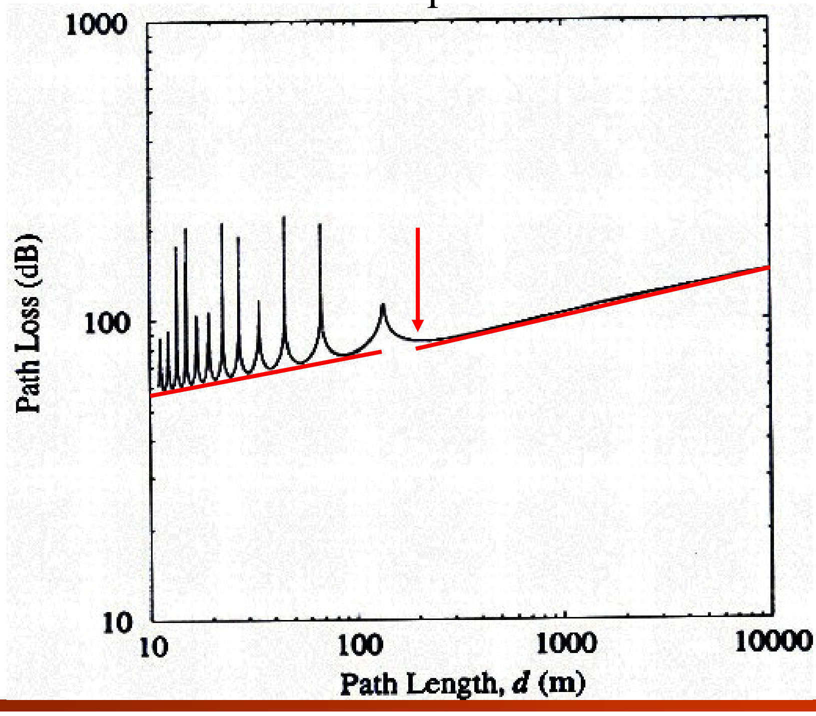
$$\mu_{\Omega_p} = \frac{k\Omega_t}{d^a(1+d/g)^b}$$

$$\begin{aligned}
\mu_{\Omega_p} &= 10\log_{10}(k\Omega_t) - 10\log_{10}(d^a(1+d/g)^b) \quad \text{(dBm)} \\
&= 10\log_{10}(k\Omega_t) - 10\log_{10} g^a - 10\log_{10}(d/g)^a - 10\log_{10}(1+d/g)^b \\
&= 10\log_{10}(k\Omega_t) - 10a\log_{10} g - 10a\log_{10}(d/g) - 10b\log_{10}(1+d/g) \\
&\approx \begin{cases} 10\log_{10}(k\Omega_t) - 10a\log_{10} d, & \text{if } d \ll g \\ 10\log_{10}(k\Omega_t) - 10a\log_{10} g - 10(a+b)\log_{10}(d/g), & \text{if } d \gg g \end{cases}
\end{aligned}$$

- When close into the BS: free-space propagation $\Rightarrow a = 2$
- At larger distance: inverse-fourth power law $\Rightarrow b = 2$

Two-Slope Model (Street Microcells) (Cont.)

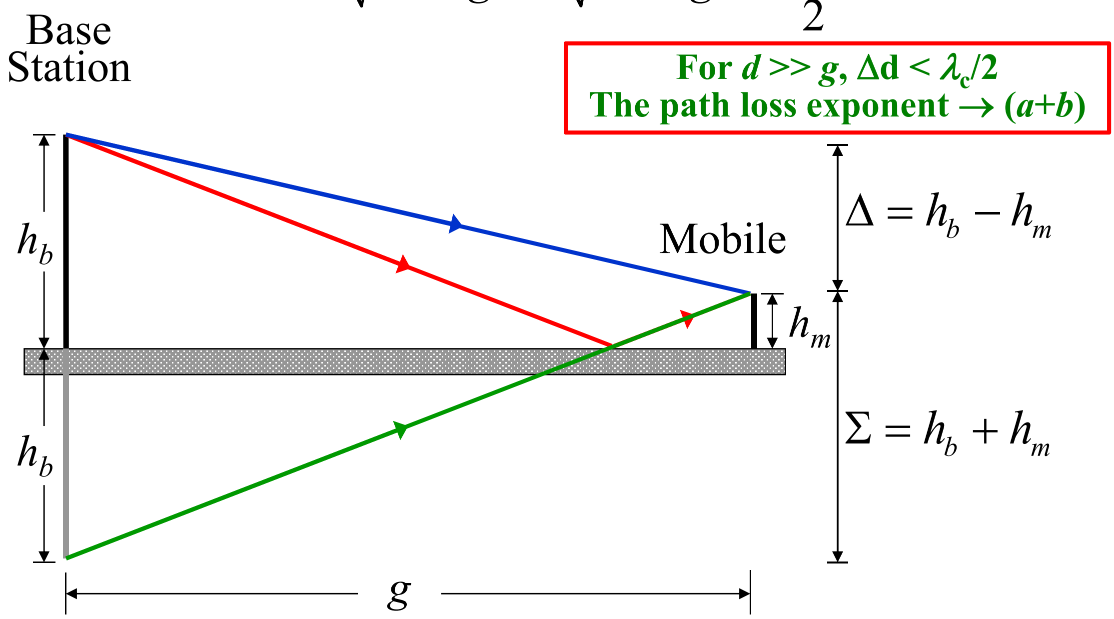
- The last local minimum of the path loss occurs when $2\pi h_b h_m / \lambda_c d = \pi/2$



Two-Slope Model (Street Microcells) (Cont.)

- The first Fresnel Zone

$$\Delta d = \sqrt{\Sigma^2 + g^2} - \sqrt{\Delta^2 + g^2} = \frac{\lambda_c}{2}$$



Two-Slope Model (Street Microcells) (Cont.)

- We set $\Sigma = h_b + h_m$ and $\Delta = h_b - h_m$
- Find the break-point g

$$\sqrt{\Sigma^2 + g^2} - \sqrt{\Delta^2 + g^2} = \frac{\lambda_c}{2}$$

$$\sqrt{\Sigma^2 + g^2} = \sqrt{\Delta^2 + g^2} + \frac{\lambda_c}{2}$$

$$\Sigma^2 + g^2 = \Delta^2 + g^2 + \lambda_c\sqrt{\Delta^2 + g^2} + (\tfrac{\lambda_c}{2})^2$$

$$\left[\Sigma^2 - \Delta^2 - (\tfrac{\lambda_c}{2})^2\right]^2 = (\lambda_c)^2(\Delta^2 + g^2)$$

$$(\Sigma^2 - \Delta^2)^2 - 2(\Sigma^2 + \Delta^2)(\tfrac{\lambda_c}{2})^2 + (\tfrac{\lambda_c}{2})^4 = (\lambda_c)^2 g^2$$

$$g = \frac{1}{\lambda_c}\sqrt{(\Sigma^2 - \Delta^2)^2 - 2(\Sigma^2 + \Delta^2)(\tfrac{\lambda_c}{2})^2 + (\tfrac{\lambda_c}{2})^4}$$

Two-Slope Model (Street Microcells) (Cont.)

- For conventional environments, break point g = 150 ~ 300m

$$g = \frac{1}{\lambda_c}\sqrt{\left(\Sigma^2 - \Delta^2\right)^2 - 2(\Sigma^2 + \Delta^2)\left(\tfrac{\lambda_c}{2}\right)^2 + \left(\tfrac{\lambda_c}{2}\right)^4}$$

- For high frequency ($\lambda_c^2 \ll (\Sigma^2 - \Delta^2)^2$):

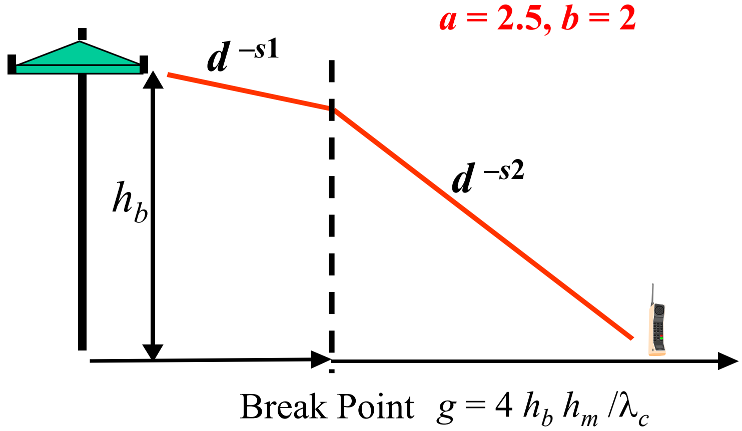
$$g \approx \frac{1}{\lambda_c}\sqrt{\left(\Sigma^2 - \Delta^2\right)^2} = \frac{\Sigma^2 - \Delta^2}{\lambda_c} = \frac{4h_b h_m}{\lambda_c}$$

Two-Slope Model (Street Microcells) (Cont.)

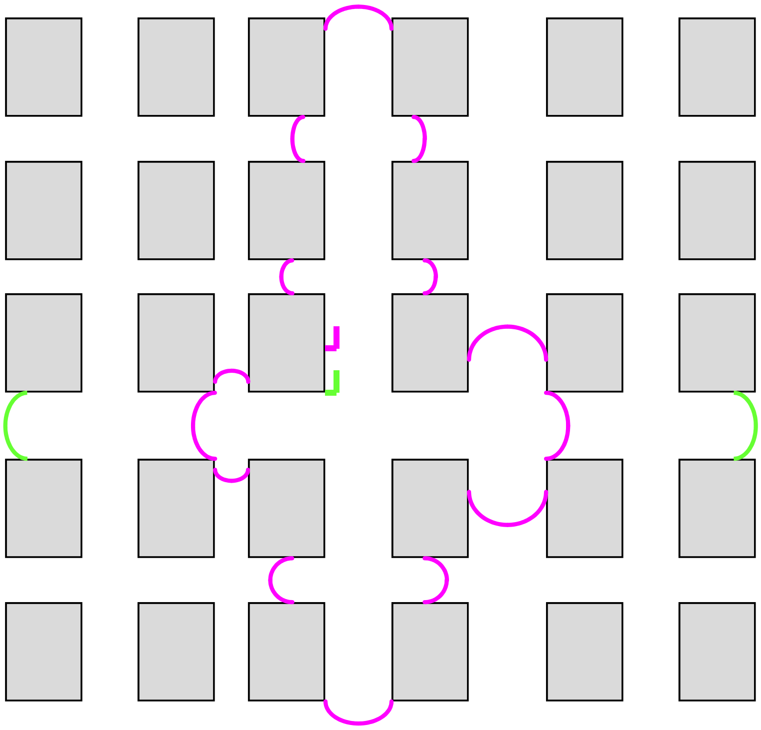
- JTC model (microcell model)
  - $d_{bp} = (4\, h_b\, h_m)/\lambda_c$, (break point)

$$L_{P(\mathrm{dB})} = \begin{cases} 38.1 + 25\log_{10}(d), & d < d_{bp} \\ 38.1 + 25\log_{10}(d_{bp}) + 45\log_{10}(d/d_{bp}), & d > d_{bp} \end{cases}$$

***a* = 2.5, *b* = 2**



Street Microcells Coverage

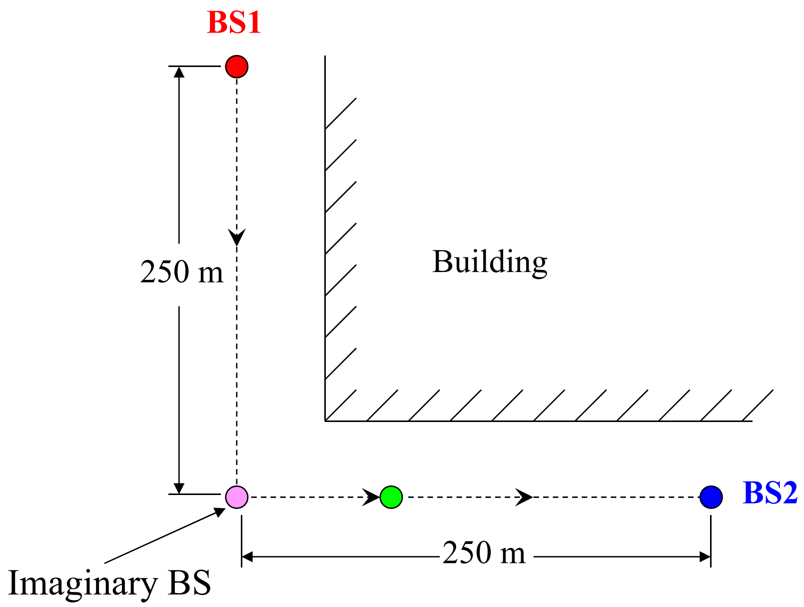


Corner Effect (Street Microcells)

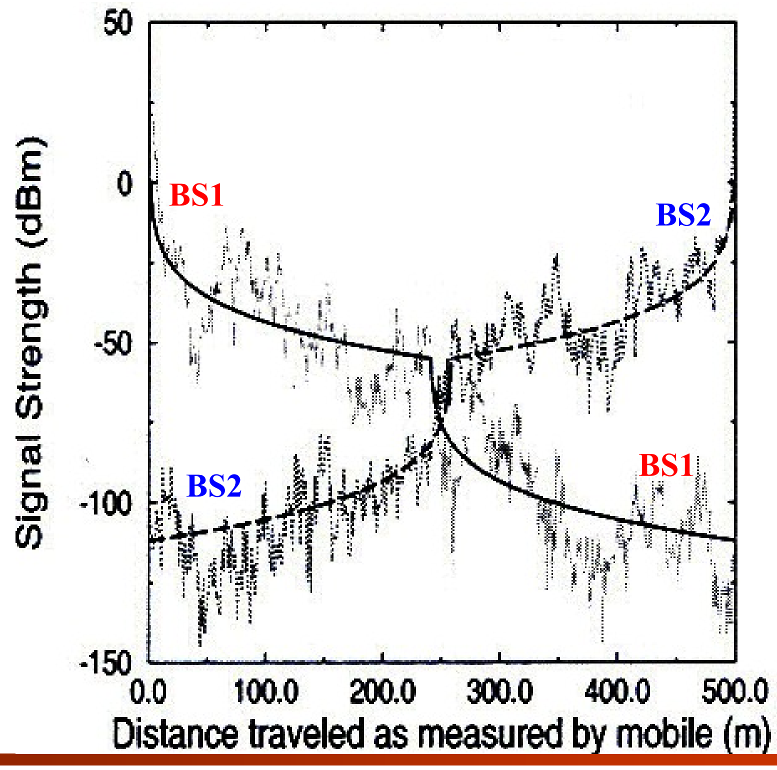
- Corner Effect: street microcells with NLOS propagation
  - The average received signal strength drops by **25~30 dB** over a distance as small as 10 m to 50 m
- The NLOS propagation is modeled as:
  - A LOS propagation from an **virtual transmitter** located at corner
  - The transmit power is equal to the received power at corner from BS

$$\mu_{\Omega_p} = \begin{cases} \dfrac{k\Omega_t}{d^a(1+d/g)^b}, & d \le d_c \\ \dfrac{k\Omega_t}{d_c^a(1+d_c/g)^b} \cdot \dfrac{1}{(d-d_c)^a(1+(d-d_c)/g)^b}, & d > d_c \end{cases}$$

Corner Effect (Street Microcells) (Cont.)



Corner Effect (Street Microcells) (Cont.)



Path Loss in Indoor Microcells

- The path loss and shadowing characteristics for indoor environments **vary greatly** from one building to another

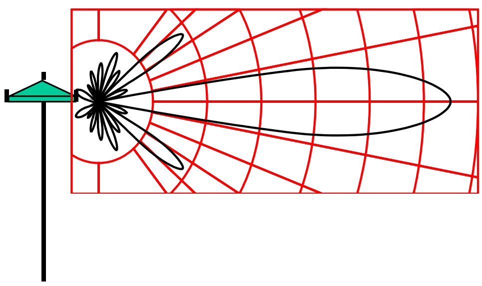
Building	Frequency (MHz)	β	σ_Ω (dB)
Retail Stores	914	2.2	8.7
Grocery Stores	914	1.8	5.2
Office, hard partition	1500	3.0	7.0
Office, soft partition	900	2.4	9.6
Office, soft partition	1900	2.6	14.1

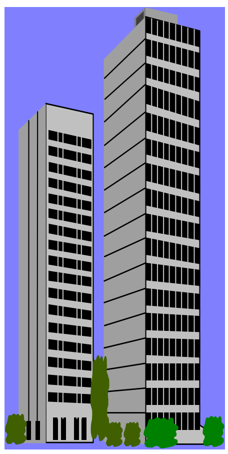
- Floor loss:
  - One floor: 15 ~ 20 dB
  - Up to 4 floors: additional 6 ~ 10 dB/floor
  - 5 or more floors: increase only a few dB for each additional floor

Path Loss in Indoor Microcells (Cont.)

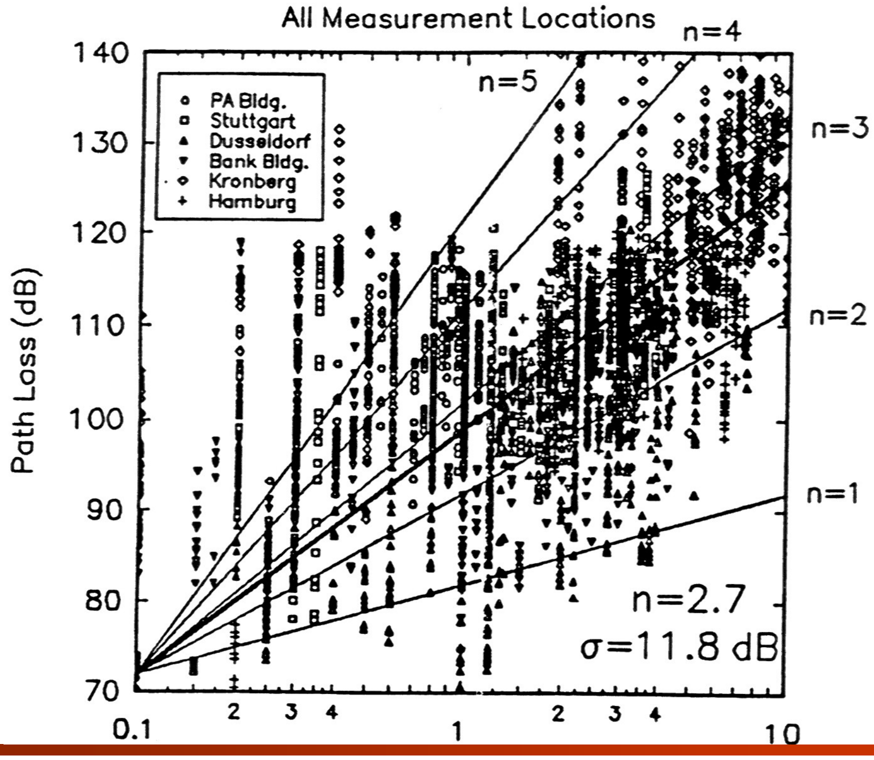
- Building penetration loss:
  - Decreases with the increase in frequency
  - Typical values: 16.4, 11.6 and 7.6 dB at 441 MHz, 896.5 MHz and 1400 MHz
  - Decreases by about 2 dB/floor from ground level up to about 9~15 floors and then **increases again**

  $\Rightarrow$ It is due to the BS antenna heights and the antenna pattern



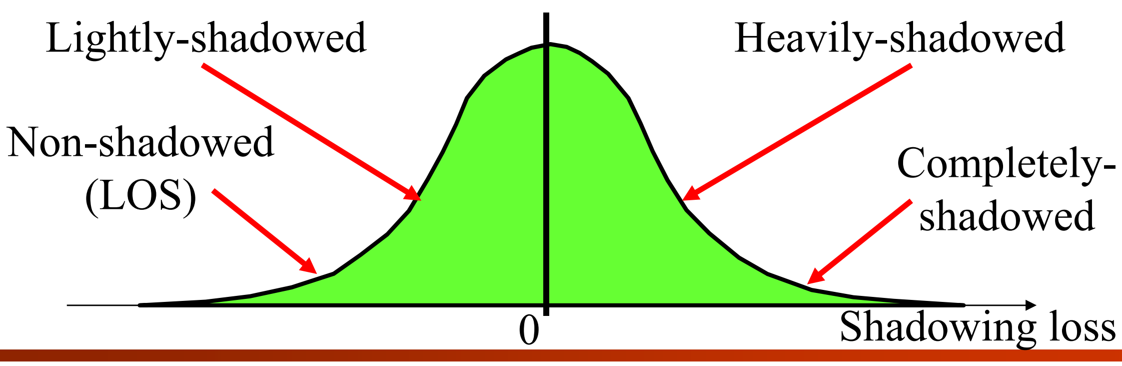


Path Loss & Shadowing



Question

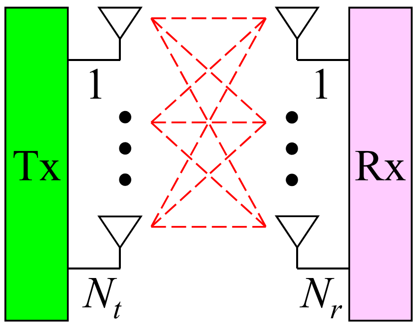
- **Question:**
  - What kind of path loss models should be applied?
  - Why the shadowing loss can be positive or negative?
- It depends on the applications (macro- or micro-cellular Sys.) and the requirements (Accuracy, Complexity, …)



MIMO Channel Models

MIMO Channel Models

- A MIMO (Multiple-Input and Multiple-Output) system is one that consists of multiple transmit and receive antennas.
- For a system consisting of N_t transmit and N_r receive antennas, the channel can be described by the $N_r \times N_t$ matrix.



Mutually Correlated!

$$\mathbf{G}(t,\tau) = \begin{bmatrix} g_{1,1}(t,\tau) & g_{1,2}(t,\tau) & \cdots & g_{1,N_t}(t,\tau) \\ g_{2,1}(t,\tau) & g_{2,2}(t,\tau) & \cdots & g_{2,N_t}(t,\tau) \\ \vdots & \vdots & & \vdots \\ g_{N_r,1}(t,\tau) & g_{N_r,2}(t,\tau) & \cdots & g_{N_r,N_t}(t,\tau) \end{bmatrix}$$

– where $g_{q,p}(t,\tau)$ denotes the time-varying sub-channel impulse response between the pth transmit and qth receive antennas.

Analytical MIMO Channel Models

- **Analytical MIMO channel models** are most often used under quasi-static flat fading conditions.
- The time-variant channel impulses $g_{q,p}(t,\tau)$ for flat fading channels can be treated as complex Gaussian random processes under conditions of **Rayleigh** and **Ricean** fading.
- The various analytical models generate the MIMO matrices as realizations of complex Gaussian random variables having specified **means** and **correlations**.
- For Ricean fading, the channel matrix can be expressed as

$$\mathbf{G} = \sqrt{\frac{K}{K+1}}\bar{\mathbf{G}} + \sqrt{\frac{1}{K+1}}\mathbf{G}_S$$

  - $\bar{\mathbf{G}}$: is the LOS or specular component (a deterministic part)
  - $\mathbf{G}_S$: is the scatter component having zero-mean (a random part)

Analytical MIMO Channel Models (Cont.)

- The **simplest** MIMO model assumes that the entries of the matrix **G** are **independent and identically distributed (i.i.d.)** complex Gaussian random variables.
  - The **rich scattering** or **spatially white** environment.
  - It simplifies the performance analysis on MIMO channels.
  - However, **in reality** the sub-channels will be **correlated**, and the i.i.d. model will lead to optimistic results.
- Define $\mathbf{g} = \text{vec}\{\mathbf{G}\} = \left[\mathbf{g}_1^T, \mathbf{g}_2^T, \cdots, \mathbf{g}_{N_t}^T\right]^T$
  - where $\mathbf{G} = [\mathbf{g}_1, \mathbf{g}_2, \cdots, \mathbf{g}_{N_t}]$ is the channel matrix
  - $\mathbf{g}$ is a zero-mean complex Gaussian random column vector of length $n = N_t \times N_r$
  - Its statistics are fully specified by the $n \times n$ covariance matrix $\mathbf{R}_{\mathbf{G}} = ½\, \mathrm{E}[\mathbf{g}\mathbf{g}^H]$

Analytical MIMO Channel Models (Cont.)

- Hence, **g** is a multivariate complex Gaussian distributed vector
  - $\mathbf{g} \sim \mathcal{CN}(\mathbf{0}, \mathbf{R_G})$
- If $\mathbf{R_G}$ is invertible, the probability density function of **g** is

$$f(\mathbf{g}) = \frac{1}{(2\pi)^n \det(\mathbf{R_G})} \exp\left[-\tfrac{1}{2}\mathbf{g}^H \mathbf{R_G}^{-1}\mathbf{g}\right], \quad \mathbf{g} \in C^n$$

- The covariance matrix $\mathbf{R_G}$ depends on the **propagation environments** and the **antenna configuration**.
- Given a covariance matrix $\mathbf{R_G}$, realizations of an MIMO channel can be generated by

$$\mathbf{G} = \text{unvec}\{\mathbf{g}\}, \quad \text{with } \mathbf{g} = \mathbf{R_G}^{1/2}\mathbf{w}$$

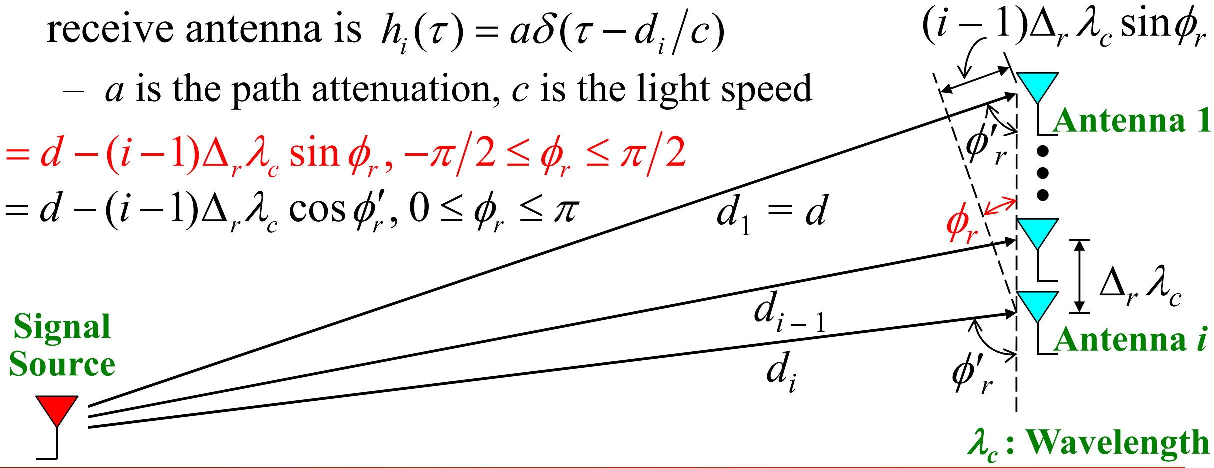
  - $\mathbf{R_G}^{1/2}$ is any matrix square root of $\mathbf{R_G}$; that is, $\mathbf{R_G} = \mathbf{R_G}^{1/2}(\mathbf{R_G}^{1/2})^H$
  - **w** is a length n vector where $\mathbf{w} \sim \mathcal{CN}(\mathbf{0}, \mathbf{I})$ (a white noise vector) for a **Rayleigh** fading channel

Angular-domain Model

- Assumed that N_t transmit antennas and N_r receive antennas are placed in **uniform linear arrays (ULA)** with the antenna separations $\Delta_t\lambda_c << d$ and $\Delta_r\lambda_c << d$. (Δ_t, Δ_r: normalized to λ_c)
- Considering a **SIMO** channel with an incidence angle ϕ_r, the time impulse response between the signal source and the i-th receive antenna is $h_i(\tau) = a\delta(\tau - d_i/c)$
  - a is the path attenuation, c is the light speed

$$d_i = d - (i-1)\Delta_r\lambda_c \sin\phi_r,\ -\pi/2 \le \phi_r \le \pi/2$$

$$d_i = d - (i-1)\Delta_r\lambda_c \cos\phi'_r,\ 0 \le \phi_r \le \pi$$



Angular-domain Model (Cont.)

- The channel gain at the i-th receive antenna is

$$g_i = a\exp(-j2\pi f_c d_i / c) = a\,\underline{\exp(-j2\pi d_i/\lambda_c)}$$
$$= a\exp(-j2\pi d/\lambda_c)\exp\left[+j2\pi\left(i-1\right)\Delta_r \sin\phi_r\right]$$

Phase difference due to different propagation distance

  – where λ_c is the carrier wavelength and f_c is the carrier frequency
- For the considered **SIMO** channel, the received signal vector can be represented as $\mathbf{y} = \mathbf{g}x + \mathbf{w}$
  – where x is the transmit signal, $\mathbf{w}$ is the channel noise vector, and

$$\mathbf{g} = ae^{-j2\pi d/\lambda_c}\begin{bmatrix} 1 \\ e^{j2\pi(d-d_2)/\lambda_c} \\ e^{j2\pi(d-d_3)/\lambda_c} \\ \vdots \\ e^{j2\pi\left(d-d_{N_r}\right)/\lambda_c} \end{bmatrix} = ae^{\psi_1}\begin{bmatrix} 1 \\ e^{j2\pi\Delta_r \sin\phi_r} \\ e^{j2\pi\times 2\Delta_r \sin\phi_r} \\ \vdots \\ e^{j2\pi\times(N_r-1)\Delta_r \sin\phi_r} \end{bmatrix} \Leftarrow$$

Array response vector for the incoming direction ϕ_r

Angular-domain Model (Cont.)

- Define the spatial frequency as $\Omega_r = \sin\phi_r$
  – $-1 \le \Omega_r \le +1$
- For the **SIMO** channel

$$\mathbf{g} = ae^{\psi_1}\begin{bmatrix} 1 \\ e^{j2\pi\Delta_r\Omega_r} \\ e^{j2\pi\times 2\Delta_r\Omega_r} \\ \vdots \\ e^{j2\pi\times(N_r-1)\Delta_r\Omega_r} \end{bmatrix}$$

- By setting the antenna separation as $\Delta_r = 1/2$

$$\mathbf{g} = ae^{\psi_1}\begin{bmatrix} 1 \\ e^{j\pi\Omega_r} \\ e^{j2\pi\Omega_r} \\ \vdots \\ e^{j(N_r-1)\pi\Omega_r} \end{bmatrix}$$

Angular-domain Model (Cont.)

- Similarly, for a **MISO** channel (transmitter: multiple antennas; receiver: single antenna) with a radiation angle ϕ_t, , the received signal can be represented as

$$y = \tilde{\mathbf{g}}^T \mathbf{x} + w$$

  - where **x** is the transmitted signal vector, w is the channel noise

- Define the spatial frequency as $\Omega_t = \sin\phi_t$

$$\tilde{\mathbf{g}} = ae^{\tilde{\psi}_1} \begin{bmatrix} 1 \\ e^{j2\pi\Delta_t\Omega_t} \\ e^{j2\pi\times 2\Delta_t\Omega_t} \\ \vdots \\ e^{j2\pi\times(N_t-1)\Delta_t\Omega_t} \end{bmatrix} \overset{\Delta_t=1/2}{=} ae^{\tilde{\psi}_1} \begin{bmatrix} 1 \\ e^{j\pi\Omega_t} \\ e^{j2\pi\Omega_t} \\ \vdots \\ e^{j(N_t-1)\pi\Omega_t} \end{bmatrix} \Leftarrow$$

Channel steering vector for the forwarding direction ϕ_t

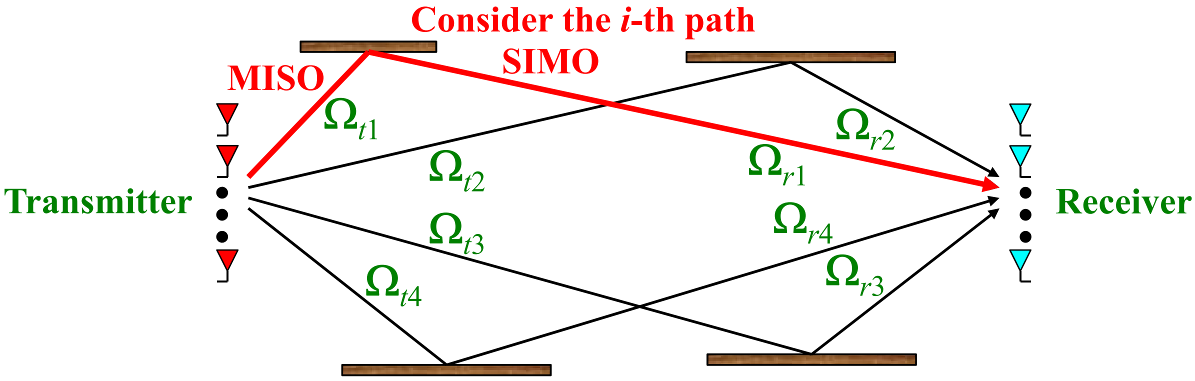
Angular-domain Model (Cont.)

- By cascading MISO and SIMO channels, a narrowband **MIMO** channel ($N_r \times N_t$ channel matrix) is represented as

$$\mathbf{y} = \mathbf{G}\mathbf{x} + \mathbf{w}$$

  - where **G** is the **spatial-domain** MIMO channel model

- Suppose there be an arbitrary number of physical paths between the transmitter and the receiver.



Angular-domain Model (Cont.)

- The i-th path has an overall attenuation of α_i, making an angle of ϕ_{ti} ($\Omega_{ti} = \sin\phi_{ti}$) for the transmit antennas and an angle of ϕ_{ri} ($\Omega_{ri} = \sin\phi_{ri}$) for the receive antennas.
- The **channel matrix G** can be represented as (sum of all paths)

$$\mathbf{G} = \sum_i \alpha_i \sqrt{N_t N_r} \exp(-j2\pi d^{(i)} / \lambda_c) \mathbf{a}_r(\Omega_{ri}) \mathbf{a}_t^H(\Omega_{ti})$$

Spatial-domain channel matrix

  – where $d^{(i)}$ is the distance between transmit antenna 1 and receive antenna 1 along **the *i*-th path**, and the **channel steering/array response vectors** are

$$\mathbf{a}_t(\Omega) \triangleq \frac{1}{\sqrt{N_t}} \begin{bmatrix} 1 \\ e^{j2\pi\Delta_t\Omega} \\ e^{j2\pi\times 2\Delta_t\Omega} \\ \vdots \\ e^{j2\pi\times(N_t-1)\Delta_t\Omega} \end{bmatrix}; \mathbf{a}_r(\Omega) \triangleq \frac{1}{\sqrt{N_r}} \begin{bmatrix} 1 \\ e^{j2\pi\Delta_r\Omega} \\ e^{j2\pi\times 2\Delta_r\Omega} \\ \vdots \\ e^{j2\pi\times(N_r-1)\Delta_r\Omega} \end{bmatrix}$$

Angular-domain Model (Cont.)

- If the antenna separation is set to $\Delta_r = \Delta_t = 1/2$, each antenna array can resolve N_r or N_t **orthogonal directions**
  – which can be regarded as a **quantized approximation** for ϕ_r, ϕ_t
  – Uniform quantization in the spatial frequency Ω_r, Ω_t
  – To form N_r, N_t orthogonal basis vectors
- The combined orthogonal basis vectors form an **IDFT** matrix

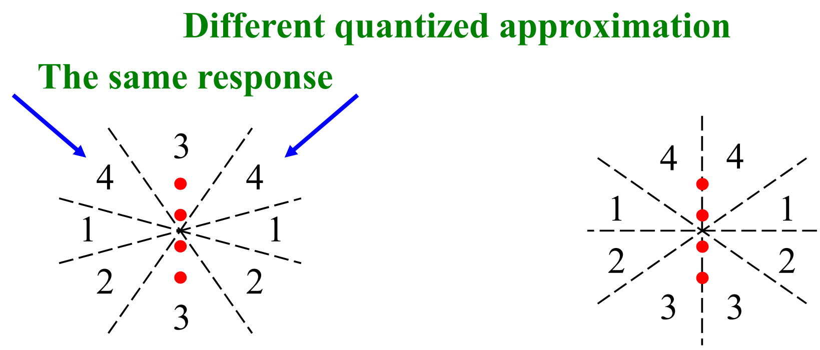
$$\mathbf{U}_t \triangleq \left[\mathbf{a}_t(0) \quad \mathbf{a}_t(2/N_t) \quad \cdots \quad \mathbf{a}_t(2(N_t-1)/N_t)\right]$$
$$\mathbf{U}_r \triangleq \left[\mathbf{a}_r(0) \quad \mathbf{a}_r(2/N_r) \quad \cdots \quad \mathbf{a}_r(2(N_r-1)/N_r)\right]$$

- If the **shortest** path is set to $d_1 = d$, the basis vectors form a **DFT** matrix

$$\mathbf{a}_t(\Omega) \triangleq \frac{1}{\sqrt{N_t}} \begin{bmatrix} 1 \\ e^{-j2\pi\Delta_t\Omega} \\ e^{-j2\pi\times 2\Delta_t\Omega} \\ \vdots \\ e^{-j2\pi\times(N_t-1)\Delta_t\Omega} \end{bmatrix}; \mathbf{a}_r(\Omega) \triangleq \frac{1}{\sqrt{N_r}} \begin{bmatrix} 1 \\ e^{-j2\pi\Delta_r\Omega} \\ e^{-j2\pi\times 2\Delta_r\Omega} \\ \vdots \\ e^{-j2\pi\times(N_r-1)\Delta_r\Omega} \end{bmatrix}$$

Angular-domain Model (Cont.)

- Note that each basis vector, $\mathbf{a}_t(2k/N_t)$ or $\mathbf{a}_r(2k/N_r)$, corresponds to **a single pair of main lobes** around the main directions.
  - Provide the best angle-domain resolution
- We can have different **quantized approximation** by applying different direction mappings



Angular-domain Model (Cont.)

- Assume that the antenna separation is $\Delta_t = \Delta_r = 1/2$, each basis vector corresponds to a **radiation** angle or an **incidence** angle.
- The transformations $\mathbf{x}^{(a)} \triangleq \mathbf{U}_t^H \mathbf{x}$ and $\mathbf{y}^{(a)} \triangleq \mathbf{U}_r^H \mathbf{y}$ correspond to transferring the coordinates of the transmitted and received signals into **the angular-domain**. ($(\cdot)^H$: Hermitian operator)
- The received signal in the angular-domain is represented as

$$\mathbf{y}^{(a)} = \mathbf{U}_r^H \mathbf{y} = \mathbf{U}_r^H \mathbf{G}\mathbf{x} + \mathbf{U}_r^H \mathbf{w}$$
$$= \mathbf{U}_r^H \mathbf{G} \mathbf{U}_t \mathbf{x}^{(a)} + \mathbf{U}_r^H \mathbf{w}$$
$$\triangleq \mathbf{G}^{(a)} \mathbf{x}^{(a)} + \mathbf{w}^{(a)}$$

  - where the **angular-domain channel matrix** is

$$\mathbf{G}^{(a)} = \mathbf{U}_r^H \mathbf{G} \mathbf{U}_t$$

  - The angular-domain noise vector is

$$\mathbf{w}^{(a)} = \mathbf{U}_r^H \mathbf{w}$$

The noise power remains the same

Angular-domain Model (Cont.)

- Hence, different physical paths (different radiation angles and/or different incidence angles) approximately contribute to different entries in the angular-domain channel matrix $\mathbf{G}^{(a)}$.
  - The angular resolution depends on N_t (N_r), and Δ_t (Δ_r)
- Based on $\mathbf{G}^{(a)} = \mathbf{U}_r^H \mathbf{G} \mathbf{U}_t$, the ($i$, j)-th element is

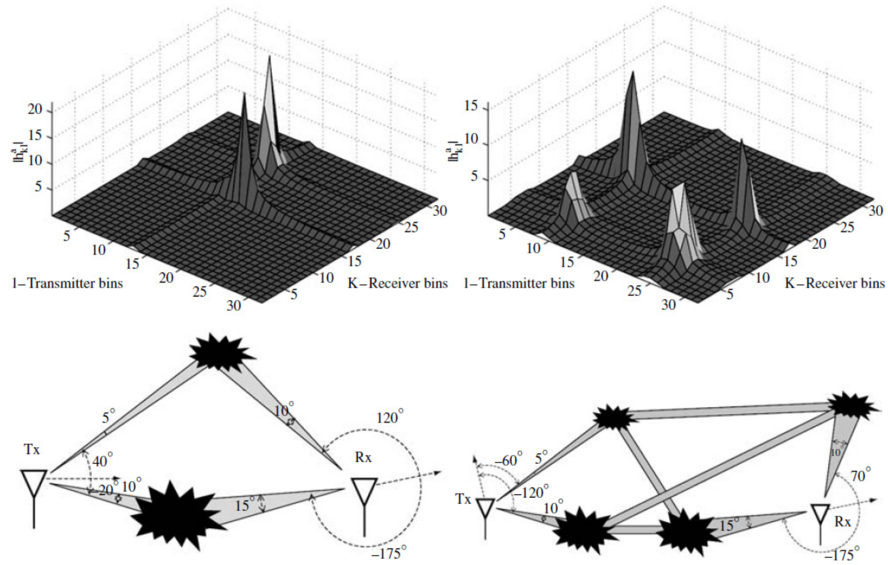
$$g_{i,j}^{(a)} = \mathbf{a}_r^H \left(2(i-1)/N_r\right) \mathbf{G} \mathbf{a}_t \left(2(j-1)/N_t\right)$$

  - which is an element contributed by the path corresponding to the j-th radiation angle and the i-th incidence angle

$$\mathbf{G}^{(a)} = \begin{bmatrix} g_{1,1}^{(a)} & g_{1,2}^{(a)} & \cdots & g_{1,N_t}^{(a)} \\ g_{2,1}^{(a)} & g_{2,2}^{(a)} & \cdots & g_{2,N_t}^{(a)} \\ \vdots & \vdots & & \vdots \\ g_{N_r,1}^{(a)} & g_{N_r,2}^{(a)} & \cdots & g_{N_r,N_t}^{(a)} \end{bmatrix}$$

Angular-domain Model (Cont.)

- For an environment with limited angular spread at the receiver and/or the transmitter, many entries of $\mathbf{G}^{(a)}$ may be zero.
  - Significantly reduce the estimation and computation complexity



Some Stochastic Channel Models

Stochastic Channel Model (SCM)

- **SCM** is a **parametric model** for the **delay spread functions**
- **Requirements for SCMs:**
  - **Completeness**: SCMs must reproduce all effects that impact on the performance of communication systems
  - **Accuracy**: SCMs must accurately describe these effects.
  - **Simplicity/low complexity**: Each effect must be described by a simple model.
- Good **SCMs** can
  - Guarantee simulation scenarios close to reality
  - Enable theoretical study of some particular system aspects and performance
  - Be used to simulate the channel in Monte Carlo simulations with acceptable computational effort

COST 207 Channel Models

Channel Models Proposed by COST

- **COST**: European COoperation in Science and Technology
- **COST-207**: Digital Land Mobile Radiocommunications (1988)
  - Channel models for GSM 900 systems
- **COST-231**: Evolution of Land Mobile Radio (including personal) Communication (1996)
  - Channel models for GSM 1800 systems
- **COST-259**: Wireless Flexible Personalized Communications (2000)
  - Channel models for DECT, UMTS and HIPERLAN 2
- **COST-273**: Towards Mobile Broadband Multimedia Networks (2005)
  - Channel models for UMTS and WLAN
  - **MIMO** channel models

COST 207 Channel Models

- Normalized delay-Doppler scattering (power) function:

$$S_n^{(\mathrm{P})}(\tau,\nu) \triangleq S^{(\mathrm{P})}(\tau,\nu)/P \Rightarrow \int S_n^{(\mathrm{P})}(\tau,\nu)\, d\tau\, d\nu = 1$$

  – where P is the total received power
- We can decompose $S_n^{(\mathrm{P})}(\tau,\nu)$ as

$$S_n^{(\mathrm{P})}(\tau,\nu) = S_n^{(\mathrm{P})}(\tau) \times S_n^{(\mathrm{P})}(\nu|\tau)$$

Normalized delay scattering function

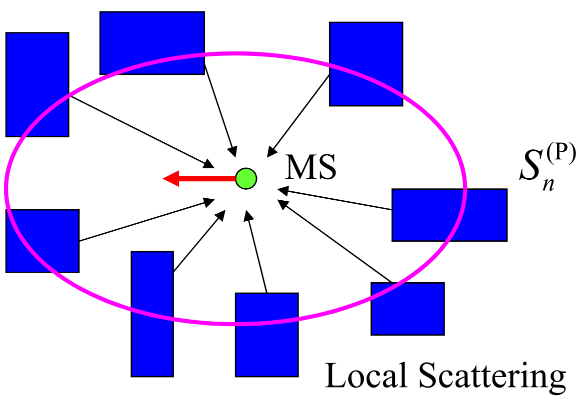
Delay-dependent normalized Doppler scattering function

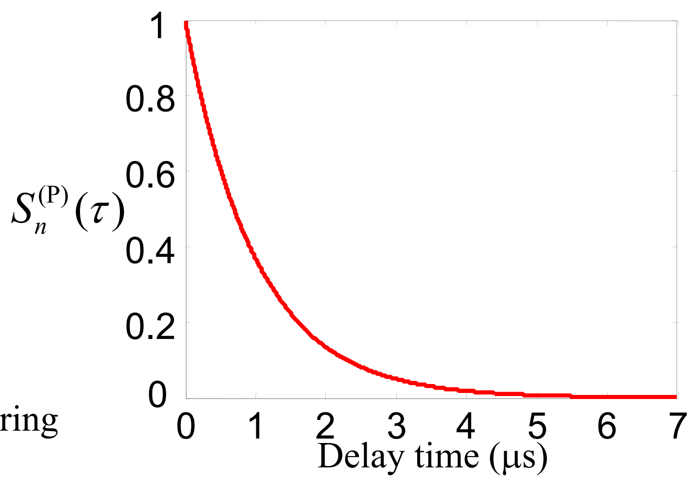
- The COST 207 models are specified by the two functions:
  – $S_n^{(\mathrm{P})}(\tau)$: scattering power of the channel in terms of the time delay τ
  – $S_n^{(\mathrm{P})}(\nu|\tau)$: scattering power of the channel in terms of Doppler frequency ν, given the time delay τ

COST 207 – Normalized Delay Scattering Fun.

- **Typical urban non-hilly area (TU)**:

$$S_n^{(\mathrm{P})}(\tau) \propto \begin{cases} \exp(-\tau); & 0 \le \tau \le 7\mu\mathrm{s} \\ 0; & \text{elsewhere} \end{cases}$$

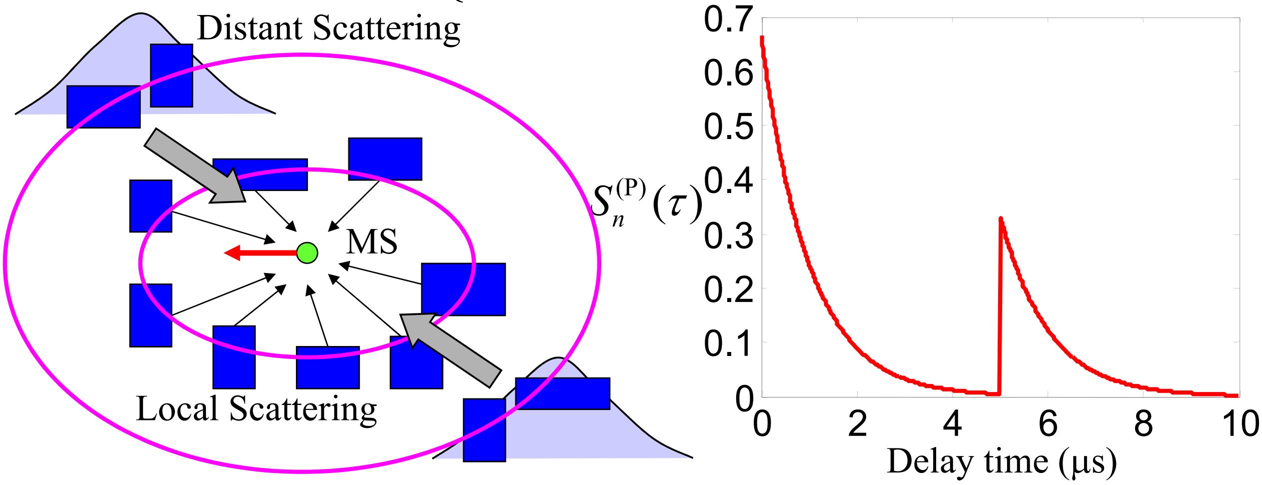




COST 207 – Normalized DS Fun. (Cont.)

- **Typical bad urban hilly area (BU)**:

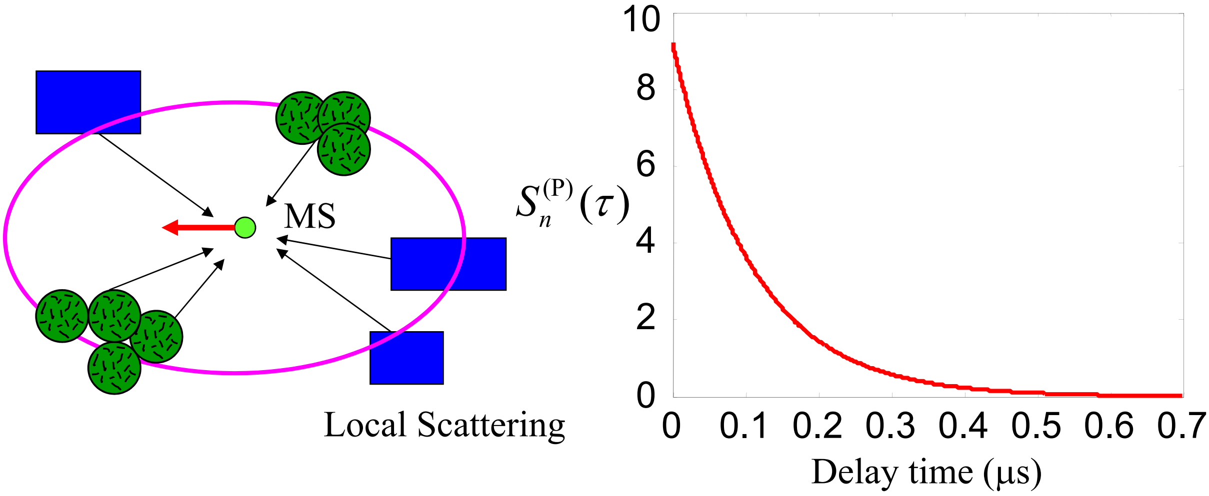
$$S_n^{(\mathrm{P})}(\tau) \propto \begin{cases} \exp(-\tau); & 0 \le \tau \le 5\mu\mathrm{s} \\ 0.5\exp(5-\tau); & 5\mu\mathrm{s} \le \tau \le 10\mu\mathrm{s} \\ 0; & \text{elsewhere} \end{cases}$$



COST 207 – Normalized DS Fun. (Cont.)

- **Typical rural non-hilly area (RA)**:

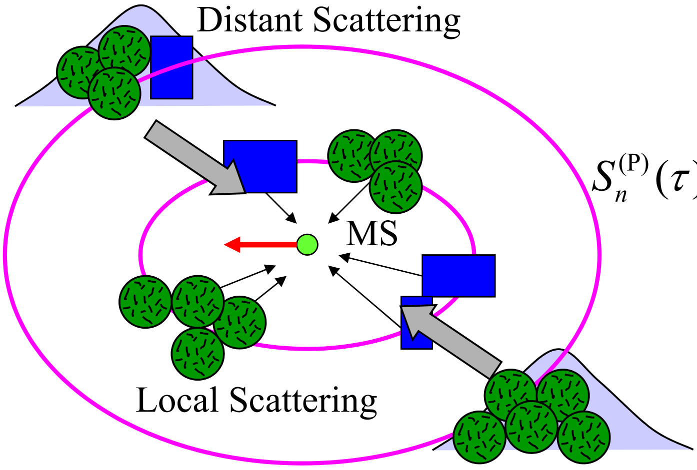
$$S_n^{(\mathrm{P})}(\tau) \propto \begin{cases} \exp(-9.2\tau); & 0 \le \tau \le 0.7\mu\mathrm{s} \\ 0; & \text{elsewhere} \end{cases}$$

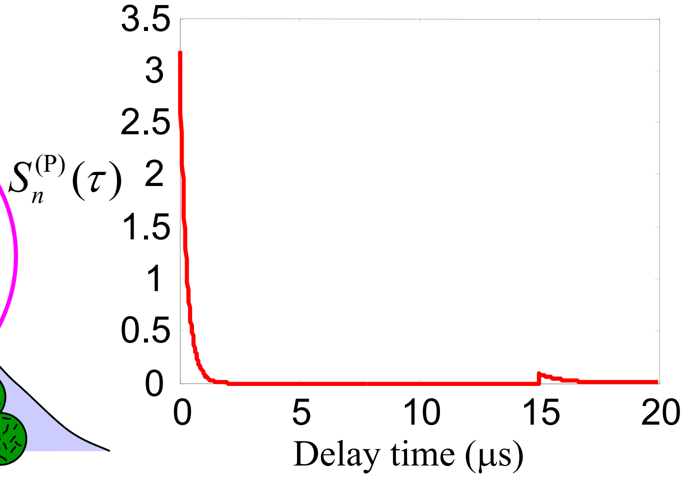


COST 207 – Normalized DS Fun. (Cont.)

- **Typical hilly terrain (HT)**:

$$S_n^{(\mathrm{P})}(\tau) \propto \begin{cases} \exp(-3.5\tau); & 0 \le \tau \le 2\mu\mathrm{s} \\ 0.1\exp(15-\tau); & 15\mu\mathrm{s} \le \tau \le 20\mu\mathrm{s} \\ 0; & \text{elsewhere} \end{cases}$$

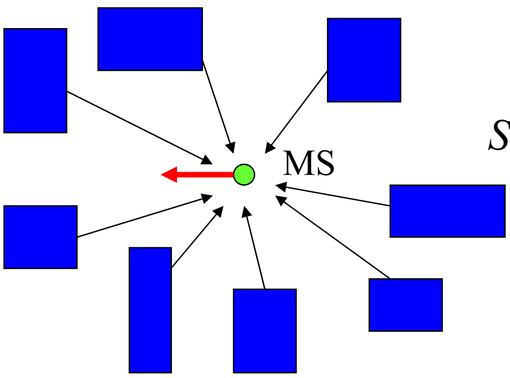


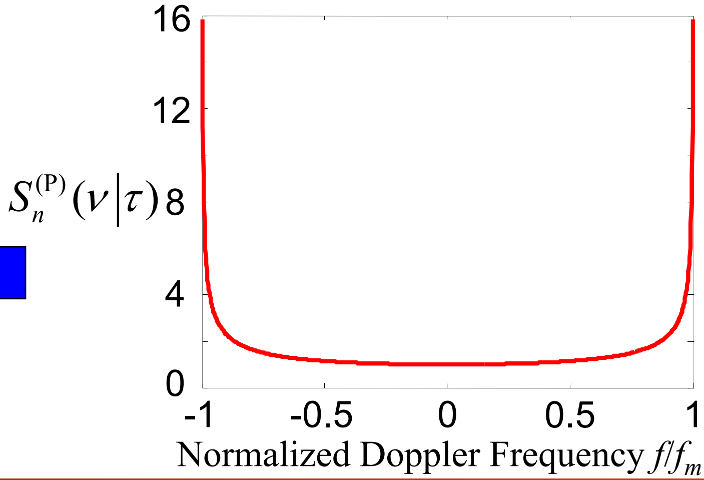


COST 207 – Normalized DS Fun. (Cont.)

- **Classical Doppler spectrum (CLASS)**: isotropic scattering ($\tau \le 0.5\mu\mathrm{s}$)

$$S_n^{(\mathrm{P})}(\nu|\tau) \propto \begin{cases} \dfrac{1}{\pi f_m} \times \dfrac{1}{\sqrt{1-(\nu/f_m)^2}}; & |\nu| \le f_m \\ 0; & \text{elsewhere} \end{cases}$$



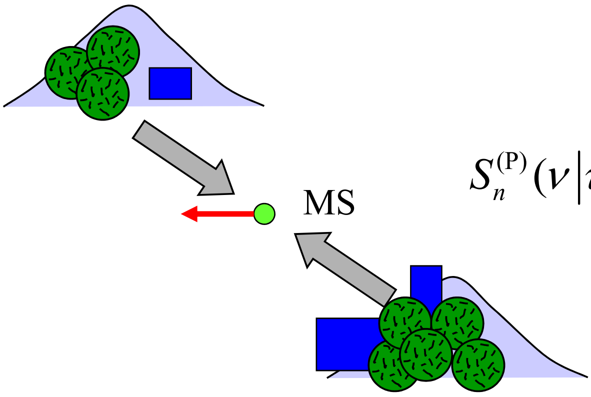


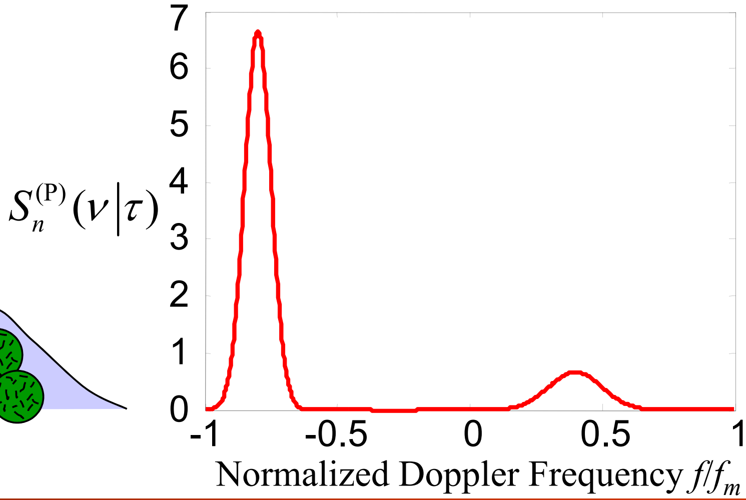
COST 207 – Normalized DS Fun. (Cont.)

- **Gaussian 1 Doppler spectrum (GAUS1)**: non-isotropic scattering ($0.5\mu s \le \tau \le 2\mu s$)

$$S_n^{(P)}(\nu|\tau) \propto G(\nu; a_1, -0.8 f_m, 0.05 f_m) + G(\nu; a_2, 0.4 f_m, 0.1 f_m)$$

$$G(\nu; a, \nu_1, \nu_2) = a \times \exp(-(\nu - \nu_1)^2 / 2\nu_2^2), \quad a_2/a_1 = -10 \text{ dB}$$



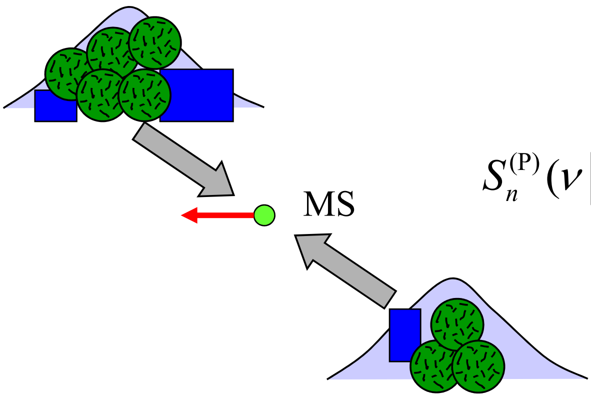


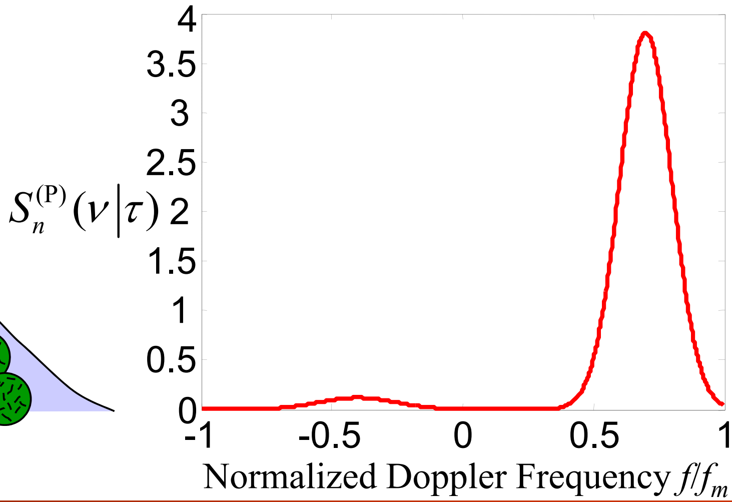
COST 207 – Normalized DS Fun. (Cont.)

- **Gaussian 2 Doppler spectrum (GAUS2)**: non-isotropic scattering ($2\mu s \le \tau$)

$$S_n^{(P)}(\nu|\tau) \propto G(\nu; a_1, 0.7 f_m, 0.1 f_m) + G(\nu; a_2, -0.4 f_m, 0.15 f_m)$$

$$G(\nu; a, \nu_1, \nu_2) = a \times \exp(-(\nu - \nu_1)^2 / 2\nu_2^2), \quad a_2/a_1 = -15 \text{ dB}$$

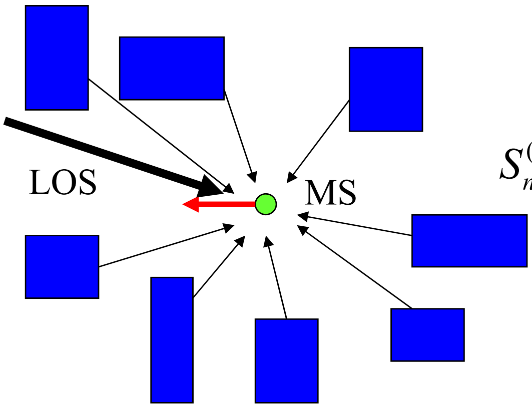


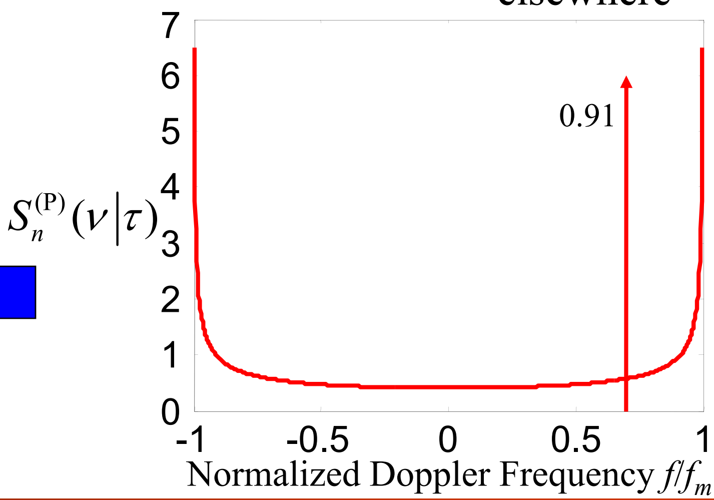


COST 207 – Normalized DS Fun. (Cont.)

- **RICE Doppler spectrum (RICE)**: isotropic scattering with LOS

$$S_n^{(\mathrm{P})}(\nu|\tau) \propto \begin{cases} \dfrac{0.41}{2\pi f_m} \times \dfrac{1}{\sqrt{1-(\nu/f_m)^2}} + 0.91\delta(\nu - 0.7 f_m); & |\nu| \le f_m \\ 0; & \text{elsewhere} \end{cases}$$



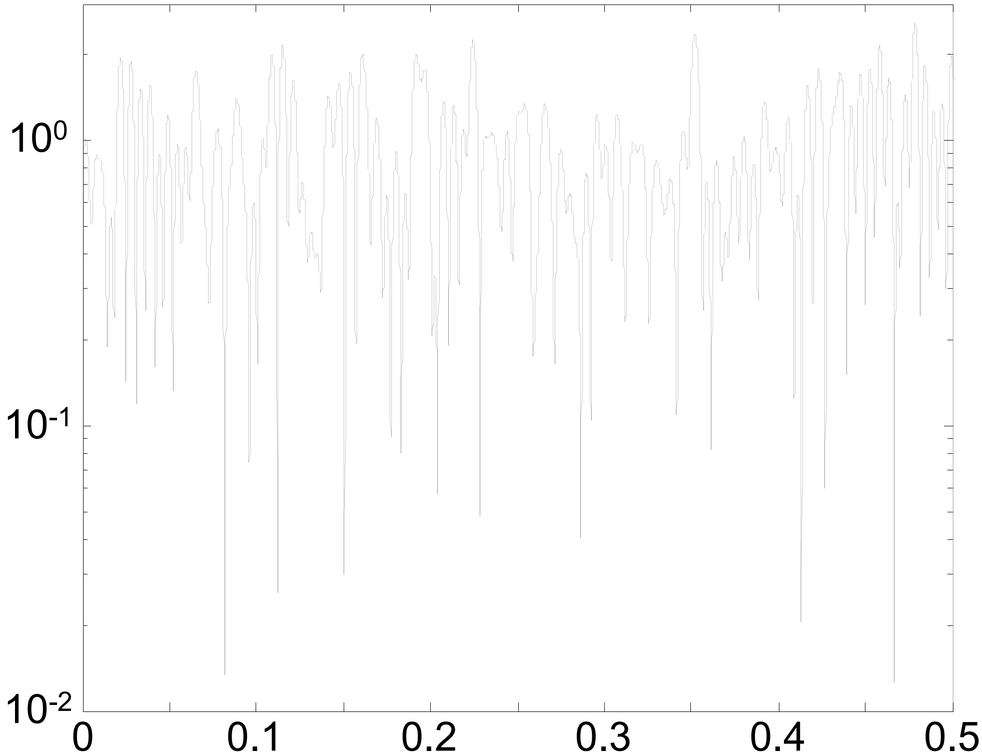


COST 207 Channel Models – TU/BU

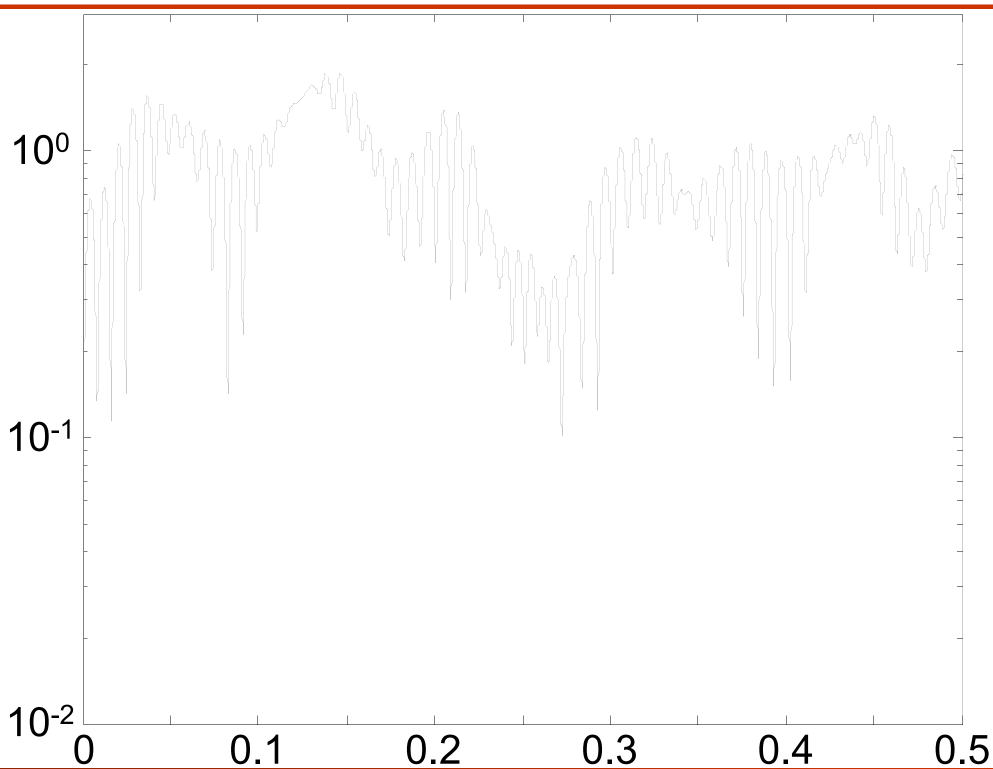
- **Typical urban (TU)** (σ_τ = 1.0 μs) and **bad urban (BU)** (σ_τ = 2.5 μs) power delay profiles

Typical Urban (TU)			Bad Urban (BU)		
Delay (μs)	Fractional Power	Doppler	Delay (μs)	Fractional Power	Doppler
0.0	0.092	CLASS	0.0	0.033	CLASS
0.1	0.115	CLASS	0.1	0.089	CLASS
0.3	0.231	CLASS	0.3	0.141	CLASS
0.5	0.127	CLASS	0.7	0.194	GAUS1
0.8	0.115	GAUS1	1.6	0.114	GAUS1
1.1	0.074	GAUS1	2.2	0.052	GAUS2
1.3	0.046	GAUS1	3.1	0.035	GAUS2
1.7	0.074	GAUS1	5.0	0.140	GAUS2
2.3	0.051	GAUS2	6.0	0.136	GAUS2
3.1	0.032	GAUS2	7.2	0.041	GAUS2
3.2	0.018	GAUS2	8.1	0.019	GAUS2
5.0	0.025	GAUS2	10.0	0.006	GAUS2

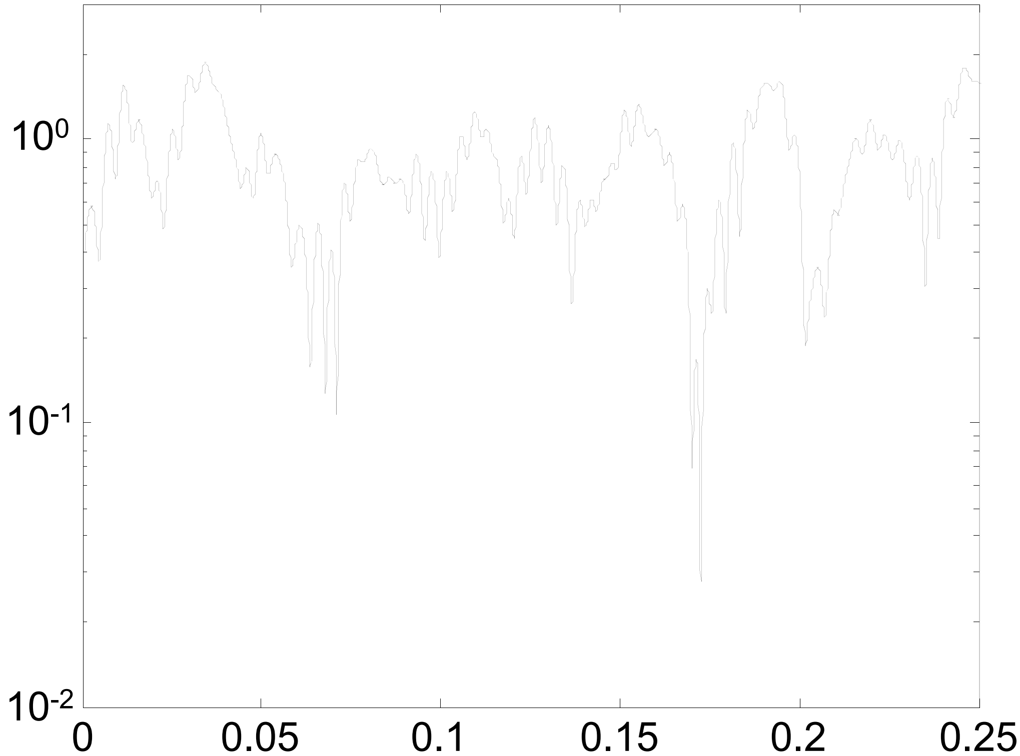
Time-domain Fading Gain – CLASS



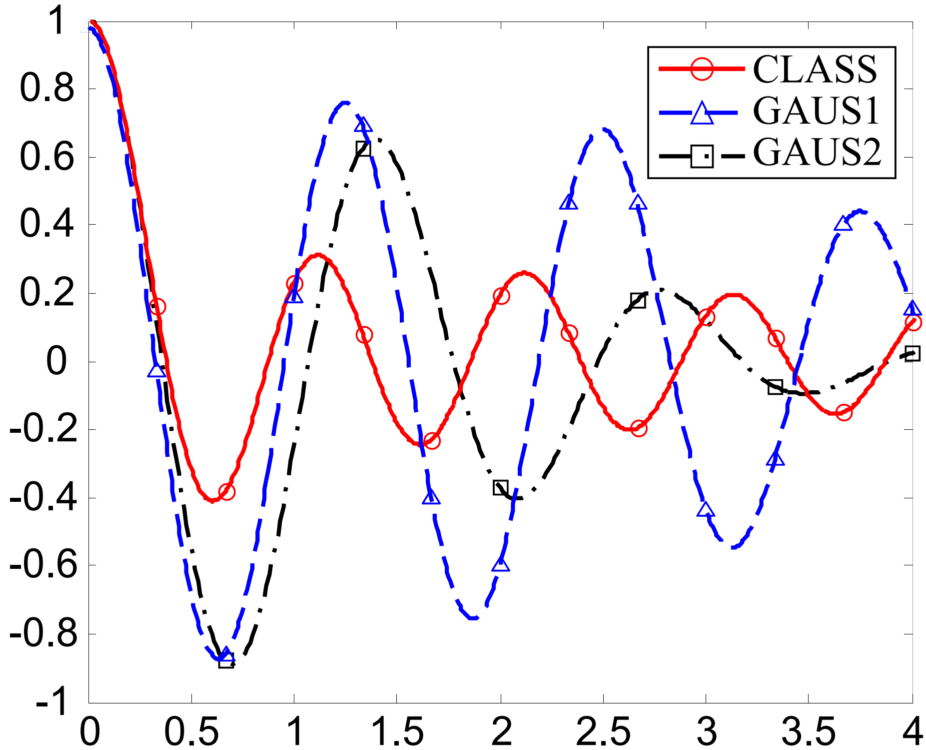
Time-domain Fading Gain – GAUS1



Time-domain Fading Gain – GAUS2



Comparison in Auto-Correlation



IEEE 802.16 Broadband Wireless Access Working Group
(Channel Models for Fixed Wireless Applications)

IEEE 802.16 Broadband Wireless Access

- **Channel Models for Fixed Wireless Applications** (2003)
  - A set of propagation models applicable to the multi-cell architecture with non-line-of-sight (NLOS) conditions is presented.
- Typically, the scenario is as follows:
  - Cells are < 10 km in radius
  - Under-the-eave/window or rooftop installed **directional** antennas (2 – 10 m) at the receiver
  - 15 – 40 m BTS antennas
  - High cell coverage requirement (80-90%)
- The wireless channel is characterized by:
  - Path loss (including shadowing), Multipath delay spread, Fading characteristics, Doppler spread, Co-channel and adjacent channel interference

IEEE 802.16 CMs – Path Loss

- For a given close-in reference distance d_0, the path loss is

$$PL_{(\text{dB})} = A + 10\gamma \log_{10}(d/d_0) + s, \quad \text{for } d > d_0$$

$$A = 20\log_{10}(4\pi d_0/\lambda), \quad \gamma = a - bh_b + c/h_b, \quad d_0 = 100m$$

  - **Category A:** hilly terrain with moderate-to-heavy tree densities
  - **Category B:** Intermediate path loss condition
  - **Category C:** mostly flat terrain with light tree densities
  - s: the shadowing effect, which follows lognormal distribution with the std. ranged between **8.2** and **10.6 dB**.

Model parameter	Terrain Type A	Terrain Type B	Terrain Type C
a	4.6	4	3.6
b	0.0075	0.0065	0.005
c	12.6	17.1	20

IEEE 802.16 CMs – Path Loss (Cont.)

- The above path loss model is based on published literature for frequencies close to **2 GHz** and for receive antenna heights close to **2 m**.
- In order to use the model for other frequencies and for **receive antenna heights** between **2 m and 10 m**, correction terms have to be included.

$$PL_{\text{modified}} = PL + \Delta PL_f + \Delta PL_h$$

  - The frequency (MHz) correction term: $\Delta PL_f = 6\log_{10}(f/2000)$
  - The receive antenna height correction term:
    - Categories A and B: $\Delta PL_h = -10.8\log_{10}(h_b/2)$
    - Category C: $\Delta PL_h = -20\log_{10}(h_b/2)$

IEEE 802.16 CMs – Multipath Delay Profile

- For directional antennas, the delay profile can be represented by **a spike-plus-exponential shape**. It is characterized by τ_{rms} (RMS delay spread) which is defined as

$$\tau_{\text{rms}} = \sqrt{\sum_j P_j \tau_j^2 - (\tau_{\text{avg}})^2}$$

- The delay profile is given by

$$P(\tau) = A\delta(\tau) + B\sum_{i=0}^{\infty} \exp(-i\Delta\tau/\tau_0)\delta(\tau - i\Delta\tau)$$

  - where A, B and $\Delta\tau$ are experimentally determined
- The delay spread model is of the following form $\boxed{\tau_{\text{rms}} = T_1 d^{\varepsilon} y}$
  - where d is the distance in km, T_1 is the median value of τ_{rms} at d = 1 km, ε is an exponent that lies between 0.5-1.0, and y is a lognormal variate.
  - 32° and 10° **directive antennas** reduce the median τ_{rms} values for omni-directional antennas by factors of 2.3 and 2.6, respectively.

IEEE 802.16 CMs – Fading Characteristics

- The narrow band received signal fading can be characterized by a **Ricean** distribution.
- A model for estimating the K-factor (in linear scale) is

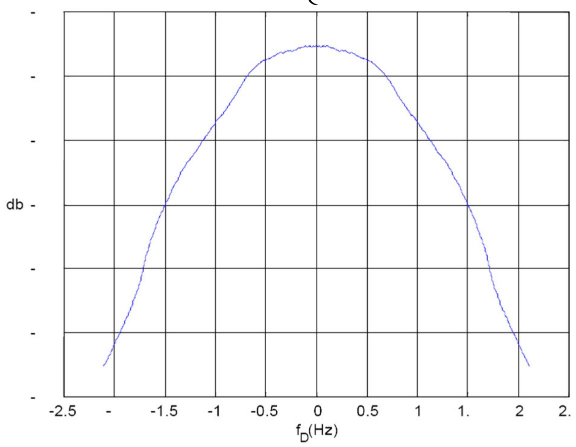
$$K = F_s F_h F_b K_o d^{\gamma} u$$

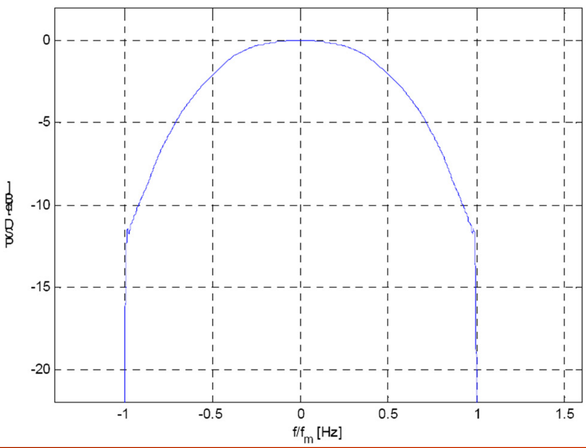
  - F_s is a **seasonal factor**, F_s = 1.0 in summer (leaves); 2.5 in winter (no leaves)
  - F_h is the receive antenna height factor, $F_h = (h/3)^{0.46}$ (h is the receive antenna height in meters)
  - F_b is the beam-width factor, $F_b = (b/17)^{-0.62}$ (b in degrees)
  - K_o and γ are regression coefficients, K_o = 10; γ = – 0.5
  - u is a lognormal variable which has zero dB mean and a std. 8 dB

IEEE 802.16 CMs – Doppler Spectrum

- In fixed wireless channels the Doppler PSD of the scattering component is mainly distributed around $f = 0$ Hz.
  - A rounded shape is used as a rough approximation

$$S(f) = \begin{cases} 1 - 1.72 f_0^2 + 0.785 f_0^4, & |f_0| \le 1 \\ 0, & |f_0| > 1 \end{cases}, \quad f_0 = \frac{f}{f_m}$$





IEEE 802.16 CMs – Antenna Gain Reduction

- The gain due to the **directivity** can be reduced because of the **scattering**.
  - The effective gain is less than the actual gain.
- Denote ΔG_{BW} as the Antenna Gain Reduction Factor.
  - **Gaussian distributed** random variable (truncated at 0 dB, i.e., $\Delta G_{\mathrm{BW}} \ge 0$) with a mean ($\mu_{\mathrm{grf}}$) and std. ($\sigma_{\mathrm{grf}}$) given by

$$\mu_{\mathrm{grf}} = -(0.53 + 0.1I)\ln(\beta/360) + (0.5 + 0.04I)(\ln(\beta/360))^2$$
$$\sigma_{\mathrm{grf}} = -(0.93 + 0.02I)\ln(\beta/360)$$

  - β: the beam-width (in degrees); $I = 1$ (winter) or $I = -1$ (summer)
- In the link budget calculation, if G is the gain of the antenna (dB), the effective gain of the antenna equals $G - \Delta G_{\mathrm{BW}}$.
  - If a 20-degree antenna is used, the mean of $\Delta G_{\mathrm{BW}} \approx 7$ dB.

IEEE 802.16 CMs – Modified SUI CMs

- **Stanford University Interim (SUI)** channel models
- The parametric view of the SUI channels is summarized in the following tables.

SUI Channels	Terrain Type	Delay Spread	Doppler Spread	K-Factor
SUI-1	C	Low	Low	High
SUI-2	C	Low	Low	High
SUI-3	B	Low	Low	Low
SUI-4	B	Moderate	High	Low
SUI-5	A	High	Low	Low
SUI-6	A	High	High	Low

IEEE 802.16 CMs – SUI - 1 Channel Model

SUI - 1	Tap 1	Tap 2	Tap 3	Units
Delay	0	0.4	0.9	μs
Power (omni ant.) 90 % K-fact. 75 % K-fact.	0 4 20	-15 0 0	-20 0 0	dB
Power (30° ant.) 90 % K-fact. 75 % K-fact.	0 16 72	-21 0 0	-32 0 0	dB
Doppler	0.4	0.3	0.5	Hz
Antenna Correlation: $\rho_{ENV} = 0.7$ **Gain Reduction Factor:** $\Delta G_{BW} = 0$ dB **Normalization Factor:** $F_{omni} = -0.1771$ dB, $F_{30°} = -0.0371$ dB	**Terrain Type:** C **Omni antenna:** $\tau_{RMS} = 0.111$ μs, overall K: $K = 3.3$ (90%); $K = 10.4$ (75%) **30° antenna:** $\tau_{RMS} = 0.042$ μs, overall K: $K = 14.0$ (90%); $K = 44.2$ (75%)			

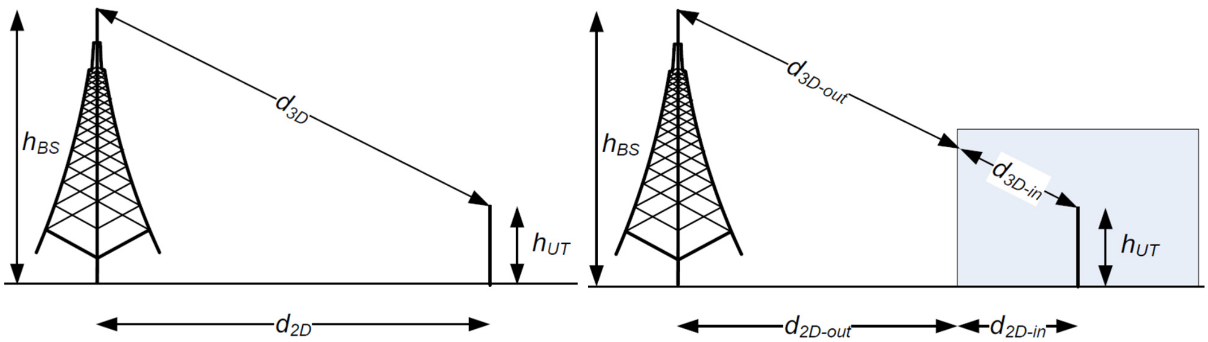
3GPP 5G Channel Models

3GPP 5G Channel Models

- Study on channel model for frequencies from **0.5 to 100 GHz**
  - 3GPP TR 38.901 V16.0.0 (2019-10)
- The channel model is applicable for **link-level** and **system-level** simulations in the following conditions:
- For system level simulations, supported scenarios are
  - Urban microcell street canyon (UMi)
  - Urban macrocell (UMa)
  - Rural macrocell (RMa)
  - Indoor hotspot office (InH)
  - Indoor factory (InF)
- Bandwidth is supported up to **10%** of the center frequency but no larger than **2GHz**.

Path Loss

- The distance definitions includes 3D distance and 2D distance
  - Outdoor UEs: d_{2D}, d_{3D}
  - Indoor UEs: $d_{2D\text{-out}}$, $d_{2D\text{-in}}$, $d_{3D\text{-out}}$, $d_{3D\text{-in}}$



d_{2D} and d_{3D} for outdoor UTs **$d_{2D\text{-out}}$, $d_{2D\text{-in}}$ and $d_{3D\text{-out}}$, $d_{3D\text{-in}}$ for indoor UTs**

Path Loss (Cont.)

- RMa

RMa	LOS	$PL_{\text{RMa-LOS}} = \begin{cases} PL_1 & 10\text{m} \le d_{2D} \le d_{BP} \\ PL_2 & d_{BP} \le d_{2D} \le 10\text{km} \end{cases}$, see note 5 $PL_1 = 20\log_{10}(40\pi d_{3D} f_c / 3) + \min(0.03h^{1.72}, 10)\log_{10}(d_{3D}) - \min(0.044h^{1.72}, 14.77) + 0.002\log_{10}(h)d_{3D}$ $PL_2 = PL_1(d_{BP}) + 40\log_{10}(d_{3D}/d_{BP})$	$\sigma_{SF} = 4$ $\sigma_{SF} = 6$	$h_{BS} = 35\text{m}$ $h_{UT} = 1.5\text{m}$ $W = 20\text{m}$ $h = 5\text{m}$ h = avg. building height W = avg. street width The applicability ranges: $5\text{m} \le h \le 50\text{m}$ $5\text{m} \le W \le 50\text{m}$ $10\text{m} \le h_{BS} \le 150\text{m}$ $1\text{m} \le h_{UT} \le 10\text{m}$
	NLOS	$PL_{\text{RMa-NLOS}} = \max(PL_{\text{RMa-LOS}}, PL'_{\text{RMa-NLOS}})$ for $10\text{m} \le d_{2D} \le 5\text{km}$ $PL_{NLOS} \ge PL_{LOS}$ $PL'_{\text{RMa-NLOS}} = 161.04 - 7.1\log_{10}(W) + 7.5\log_{10}(h) - (24.37 - 3.7(h/h_{BS})^2)\log_{10}(h_{BS}) + (43.42 - 3.1\log_{10}(h_{BS}))(\log_{10}(d_{3D}) - 3) + 20\log_{10}(f_c) - (3.2(\log_{10}(11.75h_{UT}))^2 - 4.97)$	$\sigma_{SF} = 8$	

Path Loss (Cont.)

- UMa

UMa	LOS	$PL_{\text{UMa-LOS}} = \begin{cases} PL_1 & 10\text{m} \le d_{2D} \le d'_{BP} \\ PL_2 & d'_{BP} \le d_{2D} \le 5\text{km} \end{cases}$, see note 1 $PL_1 = 28.0 + 22\log_{10}(d_{3D}) + 20\log_{10}(f_c)$ $PL_2 = 28.0 + 40\log_{10}(d_{3D}) + 20\log_{10}(f_c) - 9\log_{10}((d'_{BP})^2 + (h_{BS} - h_{UT})^2)$	$\sigma_{SF} = 4$	$1.5\text{m} \le h_{UT} \le 22.5\text{m}$ $h_{BS} = 25\text{m}$
	NLOS	$PL_{\text{UMa-NLOS}} = \max(PL_{\text{UMa-LOS}}, PL'_{\text{UMa-NLOS}})$ for $10\text{m} \le d_{2D} \le 5\text{km}$ $PL'_{\text{UMa-NLOS}} = 13.54 + 39.08\log_{10}(d_{3D}) + 20\log_{10}(f_c) - 0.6(h_{UT} - 1.5)$	$\sigma_{SF} = 6$	$1.5\text{m} \le h_{UT} \le 22.5\text{m}$ $h_{BS} = 25\text{m}$ Explanations: see note 3
		Optional $\text{PL} = 32.4 + 20\log_{10}(f_c) + 30\log_{10}(d_{3D})$	$\sigma_{SF} = 7.8$	

Path Loss (Cont.)

- UMi

UMi - Street Canyon	LOS	$PL_{\text{UMi-LOS}} = \begin{cases} PL_1 & 10\text{m} \le d_{2D} \le d'_{BP} \\ PL_2 & d'_{BP} \le d_{2D} \le 5\text{km} \end{cases}$, see note 1 $PL_1 = 32.4 + 21\log_{10}(d_{3D}) + 20\log_{10}(f_c)$ $PL_2 = 32.4 + 40\log_{10}(d_{3D}) + 20\log_{10}(f_c) - 9.5\log_{10}((d'_{BP})^2 + (h_{BS} - h_{UT})^2)$	$\sigma_{SF} = 4$	$1.5\text{m} \le h_{UT} \le 22.5\text{m}$ $h_{BS} = 10\text{m}$
	NLOS	$PL_{\text{UMi-NLOS}} = \max(PL_{\text{UMi-LOS}}, PL'_{\text{UMi-NLOS}})$ for $10\text{m} \le d_{2D} \le 5\text{km}$ $PL'_{\text{UMi-NLOS}} = 35.3\log_{10}(d_{3D}) + 22.4 + 21.3\log_{10}(f_c) - 0.3(h_{UT} - 1.5)$	$\sigma_{SF} = 7.82$	$1.5\text{m} \le h_{UT} \le 22.5\text{m}$ $h_{BS} = 10\text{m}$ Explanations: see note 4

LOS/NLOS Scenarios

- The LOS/NLOS scenarios are probabilistic with probabilities

Scenario	LOS probability (distance is in meters)
RMa	$\Pr_{\text{LOS}} = \begin{cases} 1 & , d_{\text{2D-out}} \le 10\text{m} \\ \exp\left(-\frac{d_{\text{2D-out}} - 10}{1000}\right) & , 10\text{m} < d_{\text{2D-out}} \end{cases}$
UMi - Street canyon	$\Pr_{\text{LOS}} = \begin{cases} 1 & , d_{\text{2D-out}} \le 18\text{m} \\ \frac{18}{d_{\text{2D-out}}} + \exp\left(-\frac{d_{\text{2D-out}}}{36}\right)\left(1 - \frac{18}{d_{\text{2D-out}}}\right) & , 18\text{m} < d_{\text{2D-out}} \end{cases}$
UMa	$\Pr_{\text{LOS}} = \begin{cases} 1 & , d_{\text{2D-out}} \le 18\text{m} \\ \left[\frac{18}{d_{\text{2D-out}}} + \exp\left(-\frac{d_{\text{2D-out}}}{63}\right)\left(1 - \frac{18}{d_{\text{2D-out}}}\right)\right]\left(1 + C'(h_{\text{UT}})\frac{5}{4}\left(\frac{d_{\text{2D-out}}}{100}\right)^3 \exp\left(-\frac{d_{\text{2D-out}}}{150}\right)\right) & , 18\text{m} < d_{\text{2D-out}} \end{cases}$ where $C'(h_{\text{UT}}) = \begin{cases} 0 & , h_{\text{UT}} \le 13\text{m} \\ \left(\frac{h_{\text{UT}} - 13}{10}\right)^{1.5} & , 13\text{m} < h_{\text{UT}} \le 23\text{m} \end{cases}$

Shadowing

- The log-normal shadowing fading (in dB) is characterized by a Gaussian distributed random variable with zero mean and standard deviation σ.
- Due to the slow fading process versus distance Δx, adjacent fading values are correlated.
- Its normalized autocorrelation function $R(\Delta x)$ can be described by an exponential function

$$R(\Delta x) = \exp\left(-|\Delta x| / d_{cor}\right)$$

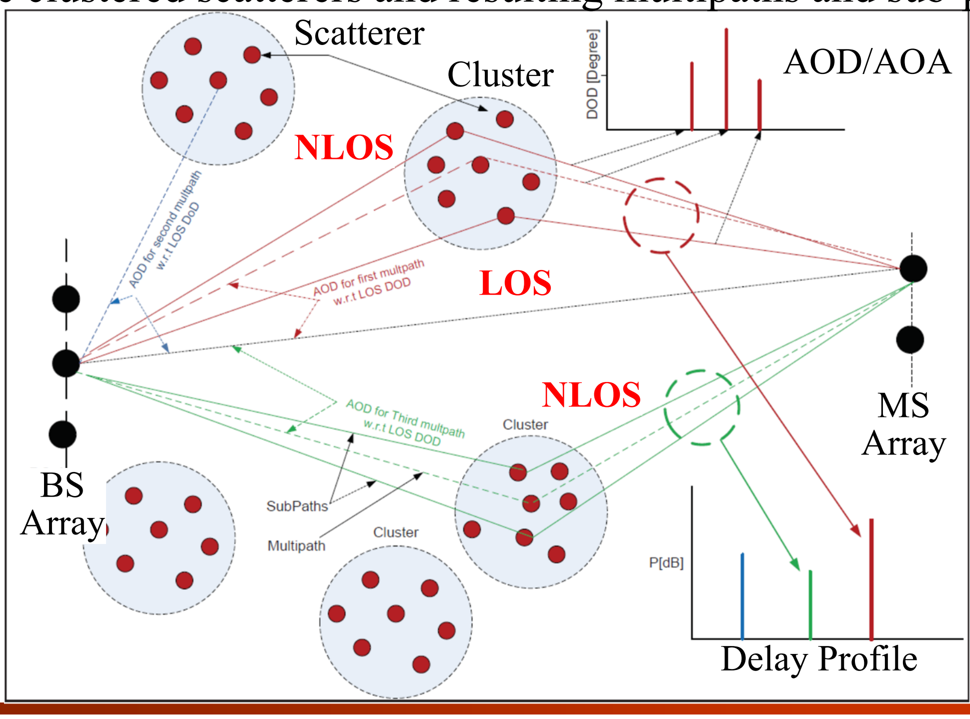
  – d_{cor}: the decorrelation length, which depends on the environment

System-level Model/Link-level Model

- The **system-level model** is a **multi-link** physical model intended for **performance evaluation**
  - Each link represents **a cell** or **a sector within a cell**.
  - An MS receives **interference** from adjacent sectors of adjacent cells.
- Each link comprises an MS and BS **MIMO antenna array**
- Propagation is via **multipaths** and **sub-paths**.
  - The excess delays of **sub-paths** are closely clustered **around the delay of their (parent) multipath**.
  - This is assumed to originate from an environment with **closely spaced clusters of scatterers**.

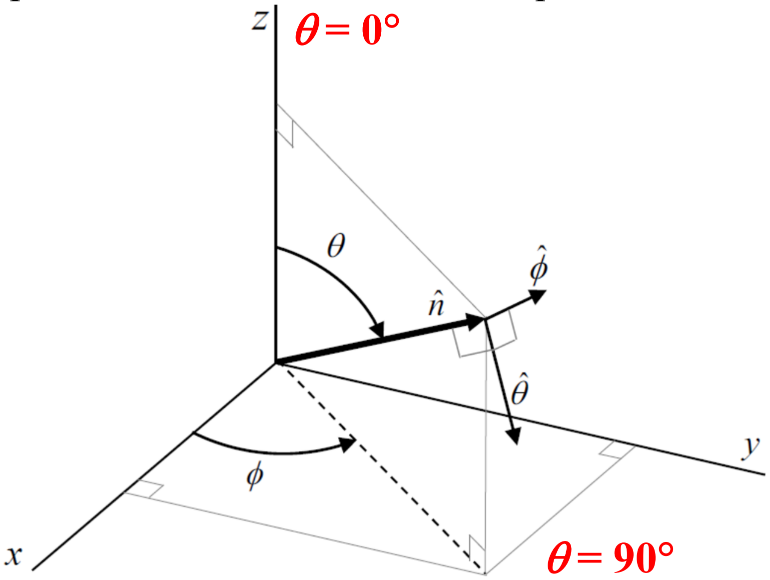
Link-level Model

- The clustered scatterers and resulting multipaths and sub-paths



Coordinate System

- The space is defined by the **zenith** angle θ ($0° – 180°$) and the **azimuth** angle ϕ ($0° – 360°$) in a Cartesian coordinate system
  - $\theta = 0°$ points to the **zenith** and $\theta = 90°$ points to the **horizon**.



Generate Cluster Delays

- Delays are drawn randomly from the exponential delay distribution $\tau'_n = -r_\tau \mathrm{DS} \ln(X_n)$
  - r_τ is the delay distribution proportionality factor
  - DS: rms delay spread
  - X_n: **uniformly** distributed within (0,1)
- For X follows the uniform distribution within (0,1), $Y = -\ln X$ is exponentially distributed with the pdf $f(y) = e^{-y}$ and $\mathrm{E}[Y] = 1$
  - τ'_n follows the **exponential distribution** with mean $r_\tau \mathrm{DS}$
- Normalize the delays by subtracting the **minimum delay** (only the **relative delays** are important) and sort the normalized delays in an ascending order

First path

$$\tau_n = \mathrm{sort}\left(\tau'_n - \min\left(\tau'_n\right)\right)$$

Generate Cluster Powers

- Cluster powers are calculated assuming a single slope **exponential** power delay profile.
- With exponential delay distribution the cluster powers are determined by

$$P'_n = \exp\left(-\tau_n \frac{r_\tau - 1}{r_\tau \mathrm{DS}}\right) \times 10^{-Z_n/10}$$

Log-normal shadowing

  - where $Z_n \sim N(0, \zeta^2)$ is the per cluster **shadowing** term in dB
- Normalize the cluster powers so that the sum of all cluster powers is equal to one, i.e., $P_n = P'_n \big/ \sum_{n=1}^{N} P'_n$
- In the case of **LOS** condition:
  - Power of the single LOS ray is: $P_{1,\mathrm{LOS}} = K_\mathrm{R} \big/ \left(K_\mathrm{R} + 1\right)$
  - The cluster powers are $P_n = \dfrac{1}{K_\mathrm{R} + 1} \times \left(P'_n \big/ \sum_{n=1}^{N} P'_n\right)$
  - K_R is the Ricean K-factor

Generate Departure and Arrival Angles

- For **the *n*-th NLOS cluster**, the AOA of the cluster $\phi_{n,\mathrm{AOA}}$ is randomly generated **centering on the AOA of LOS**
  - Depending on the **Azimuth angle Spread of Arrival** (ASA)
- The AOAs are determined by applying the inverse Gaussian function with input parameters P_n and RMS angle spread ASA

$$\phi'_{n,\mathrm{AOA}} = \frac{2(\mathrm{ASA}/1.4)\sqrt{-\ln\left[P_n / \max(P_n)\right]}}{C_\phi}$$

For a cluster closed to the LOS, it generally has a larger cluster power

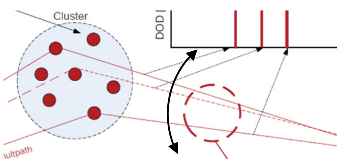
  - P_n is the power of the n-th NLOS cluster
  - C_ϕ is a scaling factor related to the total number of clusters

$$\phi_{n,\mathrm{AOA}} = X_n \phi'_{n,\mathrm{AOA}} + Y_n + \phi_{\mathrm{LOS,AOA}}$$

  - X_n: a random variable equiprobable within the set $\{1, -1\}$
  - $Y_n \sim N\left(0, \left(\mathrm{ASA}/7\right)^2\right)$ and $\phi_{\mathrm{LOS,AOA}}$ is the LOS direction

Generate Departure and Arrival Angles (Cont.)

- Generate the azimuth angle of arrival (AOA) for **the *m*-th ray in the *n*-th cluster**: $\phi_{n,m,\mathrm{AOA}} = \phi_{n,\mathrm{AOA}} + c_{\mathrm{ASA}}\alpha_m$
  - c_{ASA} is the **cluster-wise** rms azimuth spread of arrival angles
- The generation of AOD ($\phi_{n,m,\mathrm{AOD}}$), ZOA ($\theta_{n,m,\mathrm{AOA}}$), and ZOD ($\theta_{n,m,\mathrm{AOD}}$) follows a procedure similar to that for AOA.



Ray number *m*	Basis vector of offset angles α_m
1, 2	$\pm$ 0.0447
3, 4	$\pm$ 0.1413
5, 6	$\pm$ 0.2492
7, 8	$\pm$ 0.3715
9, 10	$\pm$ 0.5129
11, 12	$\pm$ 0.6797
13, 14	$\pm$ 0.8844
15, 16	$\pm$ 1.1481
17, 18	$\pm$ 1.5195
19, 20	$\pm$ 2.1551

Coupling of Rays within a Cluster

- Couple **randomly AOD** angles $\phi_{n,m,\mathrm{AOD}}$ with **AOA** angles $\phi_{n,m,\mathrm{AOA}}$ within a cluster *n*.
- Couple **randomly ZOD** angles $\theta_{n,m,\mathrm{AOD}}$ with **ZOA** angles $\theta_{n,m,\mathrm{AOA}}$ within a cluster *n*.
- Couple **randomly AOD** angles $\phi_{n,m,\mathrm{AOD}}$ with **ZOD** angles $\theta_{n,m,\mathrm{AOD}}$ within a cluster *n*.

Cross Polarization Power Ratios

- Generate the **cross polarization power ratios** (XPR) κ for each ray *m* of each cluster *n*.
  - XPR is **log-Normal distributed**

$$\kappa_{n,m} = 10^{X_{n,m}/10}$$

  - where $X_{n,m} \sim N\left(\mu_{\mathrm{XPR}}, \sigma_{\mathrm{XPR}}^2\right)$ is **Gaussian** distributed
- In general, **co-polarization** represents the polarization the antenna is intended to radiate (or receive) and **cross-polarization** represents the polarization **orthogonal to** the co-polarization
- When **cross-polarized panel array antenna** is used, different polarization antennas have different but correlated received powers

Initial Random Phases

- Draw random initial phases $\left\{\Phi_{n,m}^{\theta\theta}, \Phi_{n,m}^{\theta\phi}, \Phi_{n,m}^{\phi\theta}, \Phi_{n,m}^{\phi\phi}\right\}$ for each ray *m* of each cluster *n*
  - Four polarization combinations ($\theta\theta$, $\theta\phi$, $\phi\theta$, $\phi\phi$) (AOD/AOA pair)
  - Uniformly distributed within $(-\pi, \pi)$

Generate Channel Coefficients

- The method can be used for **drop-based evaluations**
  - Irrespective of user speed
- For the $N-2$ **weakest** (**NLOS**) (i.e., 3, 4, …, N) **clusters**, the channel coefficients, for each receiver (UE) and transmitter (BS) element pair **(u, s),** are given by

$$H_{u,s,n}^{\mathrm{NLOS}}(t)=\sqrt{\frac{P_n}{M}}\underbrace{\sum_{m=1}^{M}}_{\text{Sum of } M \text{ rays}}\underbrace{\begin{bmatrix}F_{rx,u,\theta}\left(\theta_{n,m,\mathrm{ZOA}},\phi_{n,m,\mathrm{AOA}}\right)\\ F_{rx,u,\phi}\left(\theta_{n,m,\mathrm{ZOA}},\phi_{n,m,\mathrm{AOA}}\right)\end{bmatrix}^{\mathrm{T}}}_{\text{UE antenna pattern}}\underbrace{\begin{bmatrix}\exp\left(j\Phi_{n,m}^{\theta\theta}\right) & \sqrt{\kappa_{n,m}^{-1}}\exp\left(j\Phi_{n,m}^{\theta\phi}\right)\\ \sqrt{\kappa_{n,m}^{-1}}\exp\left(j\Phi_{n,m}^{\phi\theta}\right) & \exp\left(j\Phi_{n,m}^{\phi\phi}\right)\end{bmatrix}}_{\text{Coupling between polarization components with random phases}}$$

$$\underbrace{\begin{bmatrix}F_{tx,s,\theta}\left(\theta_{n,m,\mathrm{ZOD}},\phi_{n,m,\mathrm{AOD}}\right)\\ F_{tx,s,\phi}\left(\theta_{n,m,\mathrm{ZOD}},\phi_{n,m,\mathrm{AOD}}\right)\end{bmatrix}}_{\text{BS antenna pattern}}\underbrace{\exp\left(\frac{j2\pi\mathbf{r}_{rx,n,m}^{\mathrm{T}}\cdot\mathbf{d}_{rx,u}}{\lambda_0}\right)}_{\text{Phase offset of UE antenna } u}\underbrace{\exp\left(\frac{j2\pi\mathbf{r}_{tx,n,m}^{\mathrm{T}}\cdot\mathbf{d}_{tx,s}}{\lambda_0}\right)}_{\text{Phase offset of BS antenna } s}\underbrace{\exp\left(\frac{j2\pi\mathbf{r}_{rx,n,m}^{\mathrm{T}}\cdot\mathbf{v}t}{\lambda_0}\right)}_{\text{Doppler phase shift}}$$

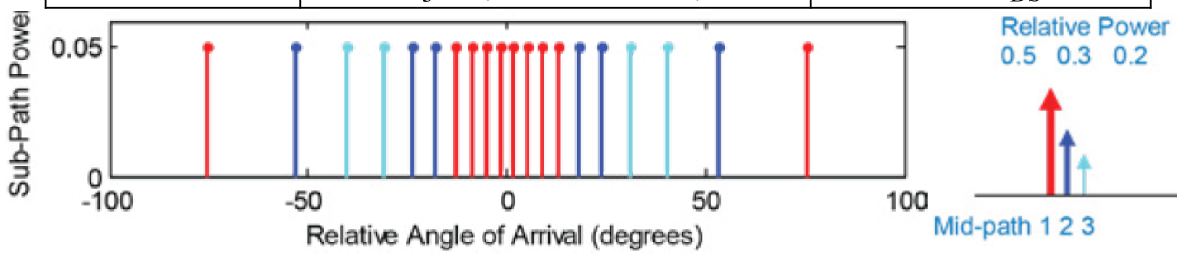
Generate Channel Coefficients (Cont.)

- rx: receiver (UE); tx: transmitter (BS)
- $F_{rx,u,\theta}$ and $F_{rx,u,\phi}$ are the **field patterns** of receive antenna element u, corresponding to θ and ϕ, respectively.
- $F_{tx,s,\theta}$ and $F_{tx,s,\phi}$ are the **field patterns** of transmit antenna element s, corresponding to θ and ϕ, respectively.
- $\mathbf{r}_{rx,n,m}$ is the **spherical unit vector** with azimuth arrival angle $\phi_{n,m,\mathrm{AOA}}$ and elevation arrival angle $\theta_{n,m,\mathrm{ZOA}}$
- $\mathbf{r}_{tx,n,m}$ is the **spherical unit vector** with azimuth arrival angle $\phi_{n,m,\mathrm{AOD}}$ and elevation arrival angle $\theta_{n,m,\mathrm{ZOD}}$
- $\mathbf{d}_{rx,u}$ is the **location vector** of receive antenna element u
- $\mathbf{d}_{tx,s}$ is the **location vector** of transmit antenna element s
- $\mathbf{v}$ is the UE **velocity vector** with speed v
- λ_0 is the **wavelength** of the carrier frequency

Generate Channel Coefficients (Cont.) – NLOS

- For the "**two strongest clusters**" (i.e., n = 1 and 2) with **NLOS**, rays are spread in delay to three sub-clusters (per cluster)
  - $\tau_{n,1} = \tau_n$; $\tau_{n,2} = \tau_n + 1.28\, c_{DS}$; $\tau_{n,3} = \tau_n + 2.56\, c_{DS}$
  - c_{DS} is the **cluster delay spread**
- Twenty rays of a cluster are mapped to sub-clusters

Sub-cluster # i	**Mapping to rays R_i**	**Delay offset $\tau_{n,1} - \tau_n$**
$i = 1$	$R_1 = \{1, 2, 3, 4, 5, 6, 7, 8, 19, 20\}$	0
$i = 2$	$R_2 = \{9, 10, 11, 12, 17, 18\}$	$1.28\, c_{\text{DS}}$
$i = 3$	$R_3 = \{13, 14, 15, 16\}$	$2.56\, c_{\text{DS}}$



Generate Channel Coefficients (Cont.) – NLOS

- Then, the channel impulse response is given by:

$$H_{u,s}^{\text{NLOS}}(\tau,t) = \sum_{n=1}^{2}\underbrace{\sum_{i=1}^{3}}_{\text{Sub-cluster}}\underbrace{\sum_{m\in R_i}}_{\text{Ray}} H_{u,s,n,m}^{\text{NLOS}}(t)\delta(\tau-\tau_{n,i}) + \sum_{n=3}^{N} H_{u,s,n}^{\text{NLOS}}(t)\delta(\tau-\tau_n)$$

(u, s): UE and BS antenna element pair

- The channel coefficients for the two strongest clusters are

$$H_{u,s,n,m}^{\text{NLOS}}(t) = \sqrt{\frac{P_n}{M}}\begin{bmatrix} F_{rx,u,\theta}\left(\theta_{n,m,\text{ZOA}},\phi_{n,m,\text{AOA}}\right) \\ F_{rx,u,\phi}\left(\theta_{n,m,\text{ZOA}},\phi_{n,m,\text{AOA}}\right)\end{bmatrix}^{\text{T}}\begin{bmatrix}\exp\left(j\Phi_{n,m}^{\theta\theta}\right) & \sqrt{\kappa_{n,m}^{-1}}\exp\left(j\Phi_{n,m}^{\theta\phi}\right) \\ \sqrt{\kappa_{n,m}^{-1}}\exp\left(j\Phi_{n,m}^{\phi\theta}\right) & \exp\left(j\Phi_{n,m}^{\phi\phi}\right)\end{bmatrix}$$

$$\begin{bmatrix} F_{tx,s,\theta}\left(\theta_{n,m,\text{ZOD}},\phi_{n,m,\text{AOD}}\right) \\ F_{tx,s,\phi}\left(\theta_{n,m,\text{ZOD}},\phi_{n,m,\text{AOD}}\right)\end{bmatrix}\exp\left(\frac{j2\pi\mathbf{r}_{rx,n,m}^{\text{T}}\cdot\mathbf{d}_{rx,u}}{\lambda_0}\right)\exp\left(\frac{j2\pi\mathbf{r}_{tx,n,m}^{\text{T}}\cdot\mathbf{d}_{tx,s}}{\lambda_0}\right)\exp\left(\frac{j2\pi\mathbf{r}_{rx,n,m}^{\text{T}}\cdot\mathbf{v}t}{\lambda_0}\right)$$

Generate Channel Coefficients (Cont.) – LOS

- For the **LOS case**, the LOS channel coefficient is given as:

$$H_{u,s,1}^{\mathrm{LOS}}(t) = \sqrt{\frac{P_n}{M}} \begin{bmatrix} F_{rx,u,\theta}\left(\theta_{\mathrm{LOS,ZOA}}, \phi_{\mathrm{LOS,AOA}}\right) \\ F_{rx,u,\phi}\left(\theta_{\mathrm{LOS,ZOA}}, \phi_{\mathrm{LOS,AOA}}\right) \end{bmatrix}^{\mathrm{T}} \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix} \begin{bmatrix} F_{tx,s,\theta}\left(\theta_{\mathrm{LOS,ZOD}}, \phi_{\mathrm{LOS,AOD}}\right) \\ F_{tx,s,\phi}\left(\theta_{\mathrm{LOS,ZOD}}, \phi_{\mathrm{LOS,AOD}}\right) \end{bmatrix}$$

$$\underbrace{\exp\left(\frac{-j2\pi d_{3\mathrm{D}}}{\lambda_0}\right)}_{\text{Phase offset due to propagation}} \exp\left(\frac{j2\pi \mathbf{r}_{rx,\mathrm{LOS}}^{\mathrm{T}} \cdot \mathbf{d}_{rx,u}}{\lambda_0}\right) \exp\left(\frac{j2\pi \mathbf{r}_{tx,\mathrm{LOS}}^{\mathrm{T}} \cdot \mathbf{d}_{tx,s}}{\lambda_0}\right) \exp\left(\frac{j2\pi \mathbf{r}_{rx,\mathrm{LOS}}^{\mathrm{T}} \cdot \mathbf{v}t}{\lambda_0}\right)$$

  - $d_{3\mathrm{D}}$ is the 3D distance between Tx and Rx
- Then, the channel impulse response is given by:

$$H_{u,s}^{\mathrm{LOS}}(\tau,t) = \sqrt{\frac{1}{K_{\mathrm{R}}+1}} H_{u,s}^{\mathrm{NLOS}}(\tau,t) + \sqrt{\frac{K_{\mathrm{R}}}{K_{\mathrm{R}}+1}} H_{u,s,1}^{\mathrm{LOS}}(t)\delta(\tau-\tau_1)$$

  - K_{R} is the Ricean K-factor

CDL Channel Models

- The CDL (**Clustered Delay Line**) channel models are defined for the full frequency range from 0.5 GHz to 100 GHz with a maximum bandwidth of **2 GHz**.
- Three CDL models, **CDL-A**, **CDL-B** and **CDL-C**, are constructed to represent three **NLOS** channel profiles
- Two models, **CDL-D** and **CDL-E**, are constructed for **LOS**
- Each CDL model can be **scaled in delay** so that the model achieves a desired **RMS delay spread**
- Each CDL model can also be **scaled in angles** so that the model achieves desired **angle spreads**

RMS delay spread and **angle spreads**
can be set according to the environment

CDL Channel Model Generation

- The following step by step procedure should be used to generate channel coefficients using the CDL models.
- Step 1: Generate departure and arrival angles
- Step 2: Coupling of rays within a cluster for both azimuth and elevation
- Step 3: Generate the cross polarization power ratios
- Step 4: Coefficient generation

Generate Departure and Arrival Angles

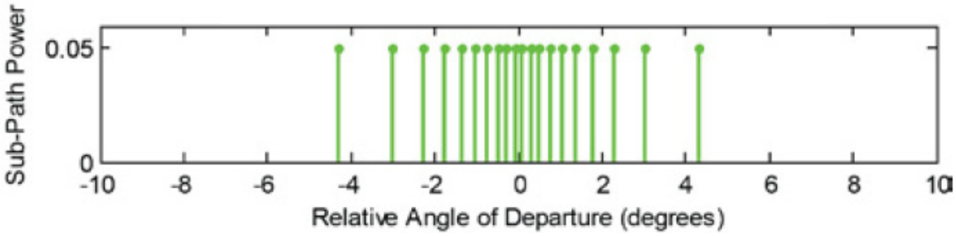
- For the n-th NLOS cluster, the AOA of the cluster $\phi_{n,\mathrm{AOA}}$ is **pre-defined** in the CDL channel models
- Generate the azimuth angle of arrival (AOA) for the m-th ray in the n-th cluster: $\phi_{n,m,\mathrm{AOA}} = \phi_{n,\mathrm{AOA}} + c_{\mathrm{ASA}}\alpha_m$
- The generation of AOD ($\phi_{n,m,\mathrm{AOD}}$), ZOA ($\theta_{n,m,\mathrm{AOA}}$), and ZOD ($\theta_{n,m,\mathrm{AOD}}$) follows a procedure similar to that for AOA.

Ray number m	Basis vector of offset angles α_m
1, 2	± 0.0447
3, 4	± 0.1413
5, 6	± 0.2492
7, 8	± 0.3715
9, 10	± 0.5129
11, 12	± 0.6797
13, 14	± 0.8844
15, 16	± 1.1481
17, 18	± 1.5195
19, 20	± 2.1551

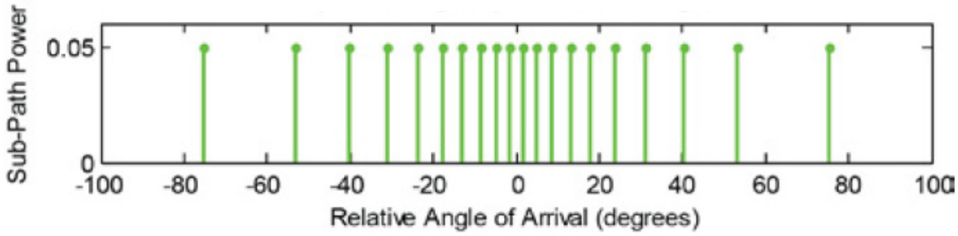
Generate Departure and Arrival Angles (Cont.)

- Each path is comprised of 20 **equal powered** sub-path components, spaced with increasing angle from the center.
- The summing of the sub-path carriers results in **Rayleigh fading** of each multipath.

Small angle spread



Large angle spread



Generate Departure and Arrival Angles (Cont.)

CDL-A

Cluster #	Normalized delay	Power in [dB]	AOD in [°]	AOA in [°]	ZOD in [°]	ZOA in [°]
1	0.0000	–13.4	–178.1	51.3	50.2	125.4
2	0.3819	0	–4.2	–152.7	93.2	91.3
3	0.4025	–2.2	–4.2	–152.7	93.2	91.3
4	0.5868	–4	–4.2	–152.7	93.2	91.3
5	0.4610	–6	90.2	76.6	122	94
6	0.5375	–8.2	90.2	76.6	122	94
7	0.6708	–9.9	90.2	76.6	122	94
8	0.5750	–10.5	121.5	–1.8	150.2	47.1
9	0.7618	–7.5	–81.7	–41.9	55.2	56
10	1.5375	–15.9	158.4	94.2	26.4	30.1
11	1.8978	–6.6	–83	51.9	126.4	58.8
12	2.2242	–16.7	134.8	–115.9	171.6	26
13	2.1718	–12.4	–153	26.6	151.4	49.2
14	2.4942	–15.2	–172	76.6	157.2	143.1
15	2.5119	–10.8	–129.9	–7	47.2	117.4
16	3.0582	–11.3	–136	–23	40.4	122.7
17	4.0810	–12.7	165.4	–47.2	43.3	123.2
18	4.4579	–16.2	148.4	110.4	161.8	32.6
19	4.5695	–18.3	132.7	144.5	10.8	27.2
20	4.7966	–18.9	–118.6	155.3	16.7	15.2
21	5.0066	–16.6	–154.1	102	171.7	146
22	5.3043	–19.9	126.5	–151.8	22.7	150.7
23	9.6586	–29.7	–56.2	55.2	144.9	156.1

Per-Cluster Parameters

Parameter	c_{ASD} in [°]	c_{ASA} in [°]	c_{ZSD} in [°]	c_{ZSA} in [°]	XPR in [dB]
Value	5	11	3	3	10

Coupling of Rays within a Cluster

- Couple **randomly AOD** angles $\phi_{n,m,\mathrm{AOD}}$ with **AOA** angles $\phi_{n,m,\mathrm{AOA}}$ within a cluster n.
- Couple **randomly ZOD** angles $\theta_{n,m,\mathrm{AOD}}$ with **ZOA** angles $\theta_{n,m,\mathrm{AOA}}$ within a cluster n.
- Couple **randomly AOD** angles $\phi_{n,m,\mathrm{AOD}}$ with **ZOD** angles $\theta_{n,m,\mathrm{AOD}}$ within a cluster n.

Cross Polarization Power Ratios

- Generate the **cross polarization power ratios** (XPR) κ for each ray m of each cluster n.
  - XPR is **log-Normal distributed**

$$\kappa_{n,m} = 10^{X/10}$$

  - where X is a pre-defined per-cluster XPR in dB
  - CDL-A: 10 dB
  - CDL-B: 8 dB
  - CDL-C: 7 dB
  - CDL-D: 11 dB
  - CDL-E: 8 dB

Gain Coefficient Generation

- All clusters are treated as "**weaker cluster**", i.e. no further sub-clusters in delay should be generated.
- Draw random initial phases $\left\{\Phi_{n,m}^{\theta\theta}, \Phi_{n,m}^{\theta\phi}, \Phi_{n,m}^{\phi\theta}, \Phi_{n,m}^{\phi\phi}\right\}$ for each ray *m* of each cluster *n*
  - Four polarization combinations ($\theta\theta$, $\theta\phi$, $\phi\theta$, $\phi\phi$) (AOD/AOA pair)
  - Uniformly distributed within $(-\pi, \pi)$
- For the **weaker** (**NLOS**) **clusters**, the channel coefficients, for each UE and BS element pair (u, s), are given by

$$H_{u,s,n}^{\mathrm{NLOS}}(t) = \sqrt{\frac{P_n}{M}} \sum_{m=1}^{M} \begin{bmatrix} F_{rx,u,\theta}\left(\theta_{n,m,\mathrm{ZOA}}, \phi_{n,m,\mathrm{AOA}}\right) \\ F_{rx,u,\phi}\left(\theta_{n,m,\mathrm{ZOA}}, \phi_{n,m,\mathrm{AOA}}\right) \end{bmatrix}^{\mathrm{T}} \begin{bmatrix} \exp\left(j\Phi_{n,m}^{\theta\theta}\right) & \sqrt{\kappa_{n,m}^{-1}} \exp\left(j\Phi_{n,m}^{\theta\phi}\right) \\ \sqrt{\kappa_{n,m}^{-1}} \exp\left(j\Phi_{n,m}^{\phi\theta}\right) & \exp\left(j\Phi_{n,m}^{\phi\phi}\right) \end{bmatrix}$$

$$\begin{bmatrix} F_{tx,s,\theta}\left(\theta_{n,m,\mathrm{ZOD}}, \phi_{n,m,\mathrm{AOD}}\right) \\ F_{tx,s,\phi}\left(\theta_{n,m,\mathrm{ZOD}}, \phi_{n,m,\mathrm{AOD}}\right) \end{bmatrix} \exp\left(\frac{j2\pi \mathbf{r}_{rx,n,m}^{\mathrm{T}} \cdot \mathbf{d}_{rx,u}}{\lambda_0}\right) \exp\left(\frac{j2\pi \mathbf{r}_{tx,n,m}^{\mathrm{T}} \cdot \mathbf{d}_{tx,s}}{\lambda_0}\right) \exp\left(\frac{j2\pi \mathbf{r}_{rx,n,m}^{\mathrm{T}} \cdot \mathbf{v}t}{\lambda_0}\right)$$