Problems

(3-1) Demonstrate that the set of integers {0,1,2,3} with addition and multiplication defined modulo-4 is not a field. Define the addition and multiplication tables which, together with this set, define a field of four elements. The polynomial $1 + D + D^2$ is primitive.

(3-2) Construct a Galois field having 32 elements and list all elements in polynomial format together with its multiplicative inverse element. The polynomial $1 + D + D^5$ is primitive.

(3-6) Let D^{10} and D^{12} represent elements of the Galois field having 16 elements and defined using the primitive polynomial $h(D) = 1 + D + D^4$.

(a) Evaluate the product $D^{10} \cdot D^{12}$ using both the polynomial representation of these elements and the power-of-D representation.

(b) Evaluate $D^{10} + D^{12}$.

(c) Evaluate D^{10}/D^{12}.

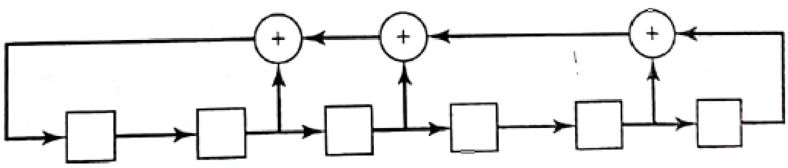
(d) Evaluate $D^{10} - D^{12}$.

(3-7) Let $g(D) = 1 + D + D^4$, $h(D) = 1 + D + D^2$, and $a(D) = 1 + D + D^2 + D^3$. Develop a shift-register configuration which generates the function

$$b(D) = \frac{h(D)}{g(D)}\, a(D)$$

and determine the circuits output as a function of time. Can this circuit be used to automate the calculation of Problem 3-6?

(3-9) Consider the feedback-shift-register configuration shown below. Determine the output of this circuit with initial condition $a(D) = 1 + D + D^2 + D^3$ and compare with the result of Problem 3-8.



PROBLEM 3-9. Feedback shift register.

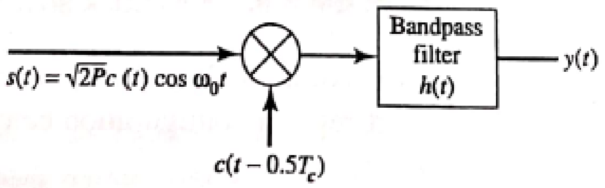
(3-12) Plot the autocorrelation function and the detailed power spectrum for the maximal-length sequence specified by $g(D) = 1 + D + D^4$ using a shift register clock rate of 1.0 kHz.

(3-16) Consider the m-sequence generator specified by the primitive polynomial 155_8. Define a shift-register configuration that will generate this sequence in the forward direction and a shift-register configuration that will generate this sequence in the reverse direction.

(3-17) The correlation-filter arrangement illustrated below is one part of a code tracking loop in a direct-sequence spread-spectrum modem. Suppose that the spreading code is an m-sequence having period N and that the bandpass filter is ideal and is specified by

$$H(\omega) = \begin{cases} 1.0 & |\omega \pm \omega_0| \leq 0.2\pi/T_c \\ 0.0 & \text{elsewhere} \end{cases}$$

Plot the ratio of the filter output power at the carrier frequency to all other power at the filter output as a function of the code period N. All of the filter output power except that at the carrier frequency is called *code self-noise.*



PROBLEM 3-17. Correlator illustrating spreading code self-noise.

(3-20) The following sequence of symbols from a spread-spectrum transmitter are received:

1 0 1 1 1 1 1 0 1 1 0 0 1

Spectral analysis of the received signal indicates that the power spectrum consists of discrete lines which are spaced at 322.6 kHz and that the spreading code rate is 10 MHz. What code generator is being used in the transmitter?

(3-24) Consider the Galois shift-register generator illustrated in Figure 3-17 with an initial load of $a(D) = 1$. Using the state-machine representation of the shift register, find the load of the shift register five cycles before and five cycles after the initial load.

(3-26) Consider a shift-register generator defined by $g(D) = 1 + D + D^3 + D^4$.

(a) Does this shift register generate a maximal-length code?

(b) Find all possible state sequences for the Fibonacci shift register for this code.

(c) Does the shift-and-add property apply to the sequences generated by this generator?